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Risk considerations in operations management

Singhal, Vinod Ramchandra, Ph.D.

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UMI

RISK CONSIDERATIONS IN OPERATIONS MANAGEMENT

by

VINOD RAMCHANDRA SINGHAL

DISSERTATION

Submitted in Partial Fulfillment

of the

Requirements for the Degree of

DOCTOR OF PHILOSOPHY

Supervised by

Uday S. Karmarkar (Chairman)
Phillip J. Lederer
G. William Schwert

William E. Simon
Graduate School of Business Administration
University of Rochester
Rochester, New York

1988

Vinod Ramchandra Singhal

CURRICULUM VITAE

Vinod Ramchandra Singhal was born on September 13, 1957 in Tatanagar, India. In 1979, he received his bachelors degree in mechanical engineering from Birla Institute of Technology and Science, Pilani, India. In 1981, he received his masters in business administration from Indian Institute of Management, Ahmedabad, India. After obtaining his MBA, he worked for a year as a management consultant with A. F. Ferguson & Co., Bombay, India. He joined the Ph.D. program in operations management at the Simon School of Business, the University of Rochester in the fall of 1982. In 1985, he received a masters of science in operations research from the University of Rochester. Since August of 1986, he has been working as a Senior Research Scientist at General Motors Research Laboratories, Warren, Michigan.

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ABSTRACT

The risk of the firm is directly affected by many resource acquisition and allocation decisions made in operations management. Yet, if one examines the procedures used by firms or described in operations management literature, there is little evidence of this relationship. This thesis is concerned with the relation between manufacturing decisions and the risk of the firm. Two basic questions are addressed: First, how should the notion of business risk enter into manufacturing decisions at the resource allocation and acquisition levels? Second, how would manufacturing decisions be affected by risk considerations? Three separate topics that address these questions are considered.

The first topic considers the effect of inventories on the risk of the firm. The main conclusions are: (1) the risk of the firm is an increasing function of the inventory level, (2) the value maximizing inventory level is a decreasing function of the riskiness of demand, and (3) the results from an empirical study weakly support the hypothesis that firms holding higher inventories are more risky.

The second topic is concerned with the effect of risk aversion of the owners of the firms on the equilibrium price and service level in a competitive market where service is measured by the probability of product availability. It is shown that more risk averse owners stock less and provide lower service levels. It is argued that more risk averse owners would dominate the low price-service market segments whereas less risk averse would dominate the high price-service market segments.

The third topic focuses on the effect of the cost structure of new technologies on the risk of the firm. New technologies require a higher initial investment than conventional technologies, but have lower fixed operating costs per period (excluding depreciation) and lower variable costs per unit when compared to conventional technology. Models that consider the implications of the differences in the cost structure of these technologies on the appropriate discount rates are developed. It is shown that, in many cases, the appropriate discount rate for evaluating technologies solely on the basis of costs should be lower than when both revenues and cost are considered.

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CHAPTER 1.

Introduction and Summary

In recent years, manufacturing managers have faced a rapidly changing environment. The changes have been both within manufacturing itself as well as in the market for the products that are to be manufactured. In many product markets, competition has intensified to the point where manufacturing costs often become a key competitive issue. Furthermore, there are important marketing issues such as product-mix flexibility, product quality and customer response time that are directly dependent on manufacturing capability. Together with rapid changes in the product markets in which a firm competes, manufacturing technology is also undergoing rapid changes. New developments in manufacturing are occurring in hard technology having to do with equipment (robotics, automated material handling, and flexible manufacturing systems) as well in the softer technology relating to procedures, organization and information processing. The manufacturing manager is faced with a wide spectrum of different equipment and system choices, involving substantial levels of investments and risks, with strategic implications for the firm as a whole. It is thus necessary to understand how manufacturing decisions affect the business risk of the firm.

The issue of business risk and how it is affected by manufacturing decisions has not been adequately addressed in Operations Management literature. Most of the existing resource allocation and acquisition decisions models analyze these decisions on the basis of expected values while ignoring the effect on risk of these decisions. Some attempt has been made to incorporate risk by modeling these decisions in present value terms using discount rates. But, the basic issue of how manufacturing decisions affect the risk and, hence, the discount rates is not addressed. More often than not, it is assumed that the discount rates are given and fixed. This is also reflected in capital budgeting procedures which use "hurdle rate" techniques that do not allow for variation in discount rates to reflect the risk of investment. This is not a severe problem when the markets for a firm's products are stable and when the technology choices available are so few that the choice of technology is obvious. As discussed earlier, this situation is changing rapidly due to the introduction of new technologies and intensification of competition. In fact, the risk of the firm cannot be taken as given but is influenced by the choice of technology and other manufacturing decisions.

This thesis is concerned with the relation between manufacturing decisions and the risk of the firm. Part of the importance of understanding this relation arises from the need to manage manufacturing activities in a competitive, rapidly changing market. Furthermore, the market value of the firm is directly affected by the risk of the firm. Given that the objective of managers is to maximize the market value, it is important to understand how operating decisions affect the risk and, hence, the market value of the firm. Two basic questions are addressed here: First, how should the notion of business risk enter into manufacturing decisions at the resource allocation and acquisition levels? Second, how would manufacturing decisions be affected by risk considerations? Three separate topics that address these questions are considered. These topics are: (1) the effect of inventories on the risk of the firm, (2) the effect of the risk aversion of owners of the firm on the equilibrium price-service levels in a competitive market, and (3) the effect of the cost structure of new technologies on the risk of the firm.

Each topic is discussed in a separate chapter. The first part of each chapter introduces the topic studied, discusses why it is important, and relates it to the field of operations management. The methods and techniques used, the analysis, and the results are discussed in the latter part of a chapter. Each chapter closes with a summary.

Chapter 2, "Inventories, Risk, and the Value of the Firm," uses the Sharpe (1964)-Lintner (1965) Capital Asset Pricing Model and the Black-Scholes(1973) Option Pricing Model to link the inventory decisions of a firm with the firm's risk. Whereas most stochastic inventory models in management science literature derive optimal inventory levels assuming that the risk, and hence, the opportunity cost of capital is invariant with the level of inventory, this chapter shows that the opportunity cost of capital for investment in inventories is an increasing function of the inventory level. Optimality conditions for the inventory level analogous to the "Newsboy" problem are derived. The value maximizing inventory level is a decreasing function of the riskiness of demand, where the risk of the demand is measured by its covariability with the return on the portfolio that consists of all risky assets in the market. Holding inventory creates operating leverage similar to the leverage from the commitment to fixed manufacturing costs. The higher the level of inventory, the more levered is the firm, and hence, more risky. The results from an empirical study weakly support the hypothesis that firms holding higher inventories are in fact more risky. Finally, it is shown that the benefits of investments that reduce inventories are not just in the savings in inventory holding costs but also the benefits from lower risks of holding inventories and from higher service levels. These benefits can be quantified and reflected in procedures for evaluating such investments.

Chapter 3, "Risk Aversion, Inventories, and Service Levels: An Equilibrium Analysis," is concerned with the effect of risk aversion of the owners of the firms on the equilibrium price and service level in a competitive market where service is measured by the probability of product availability. It extends Gould's (1978) equilibrium model developed under the assumption of risk neutrality. It develops a model of market equilibrium where firms face stochastic demand, sell a single product, know their cost functions with certainty, and the owners maximize the expected utility of profits. The existence of an equilibrium is demonstrated. It is shown that more risk averse owners stock less and provide lower service levels. I argue that more risk averse owners would dominate the low price-service market segments whereas less risk averse would dominate the high price-service market segments.

Chapter 4, "Financial Justification of New Technologies," addresses the issue of business risk of investment decisions in new manufacturing technologies such as flexible automation, robotics, automated material storage and handling, and the computer integration of manufacturing systems. Existing capital budgeting procedures typically use "hurdle rate" techniques which do not allow for the variation in the risk of new technologies. Although there are many factors that affect the risk of firm, this chapter focuses on the effect of the cost structure of new technologies on the risk of the firm. Examples of successful implementation of new technologies are used to show that the cost structure of new technologies is significantly different from that of conventional technologies. In particular, new technologies require a higher initial investment than conventional technologies, but have lower fixed operating costs per period (excluding depreciation) and lower variable costs per unit when compared to conventional technology. Models that consider the implications of the differences in the cost structure of these technologies on the appropriate discount rates are developed. It is also shown that the appropriate discount rate for evaluating technologies solely on the basis of costs is different from the discount rate when both revenues and costs are considered. In many cases, the discount rates when only costs are considered should be lower than when both revenues and cost are considered.

CHAPTER 2.

Inventories, Risk, and the Value of the Firm

2.1. Introduction

Firms often use mathematical models to choose inventory levels by balancing setup or ordering costs and stockout costs with the cost of holding inventory. The cost of holding inventory includes not only storage cost and the cost of spoilage or obsolescence, but also the opportunity cost of capital, that is to say, the rate of return offered by other, equivalent-risk investment opportunities. This chapter examines a basic question: How do the inventory decisions of a firm affect the risk, and hence, the opportunity cost of capital of the firm? The answer to this question not only has implications for the optimal level of investment in inventories, but also for the justification of investments that alter the characteristics of production-distribution systems. The three examples discussed below illustrate this point.

A commonly used measure of system performance of base-stock systems is the fraction of demand filled from stock. This measure is often called the service level of the system. Consider two alternative base stock levels, one that provides a 95% service level and the other a 90% service level. The high service level alternative is more risky, in the sense that the variance of the net cash flow is higher, since there is a higher chance of being left with unsold stock. The expected net cash flows from of the two alternatives are also different. The high service level alternative offers higher expected net cash flows since the probability of stockout is low. Both alternatives have different risk and expected net cash flow characteristics. Using the same opportunity cost of capital (or holding cost) to choose between the two can lead to an incorrect decision.

Consider a firm that has a replenishment lead time of four weeks and holds safety stock to meet uncertain demand during the replenishment time for the supply of products. Suppose that at some cost the firm can reduce the replenishment lead time to one week. With a lead time of one week the amount of safety stock necessary to provide a given service level will decrease. Most inventory models evaluate this opportunity to reduce lead time by comparing the savings in inventory carrying cost with the cost of reducing lead time. More often than not, the impact of reduced lead time on the risk of holding safety stock and on the firm's revenues are ignored. With reduced lead time the firm can provide the same service level with less safety stock, thereby lowering the risk of carrying safety stock. Futhermore, the optimal policy could now be to increase service level, which will have a positive effect on revenues. Such benefits can be substantial but are rarely considered when justifying investments that reduce lead time.

Investing in technologies that reduce setup costs is another example where the risk of holding inventories is reduced. One of the major advantages of new manufacturing technologies like Flexible Manufacturing Systems(FMS) and Robotics is that setup time and costs are low, making it economical to manufacture parts or products in small batches (e.g, see Ayres and Miller 1981, Thompson and Paris

1982, and Bylinsky 1983). In an environment where competition has intensified, with markets and products changing rapidly, the advantages of using FMS and Robotics are not only the reduction in holding costs but also the significant reduction in the risk of holding inventories, especially that of obsolescence.

The above three examples illustrate how inventories affect the risk of the firm. Unfortunately, existing inventory models in management science literature have not adequately addressed the question of the risk of inventories. In most models the risk of holding inventories is separated from optimal investment in inventories. The emphasis has been on deriving optimal inventory policies by minimizing an inventory cost expression which is the sum of ordering costs, holding costs and stockout costs. The holding cost includes the opportunity cost of capital which should reflect the risk of holding inventories, but most Operations Management texts do not address this point. The point that is missed is that the level of inventory determines the risk, and therefore, the opportunity cost of capital. Assuming that holding costs do not change with the level of inventory assumes that the risk of holding inventory is invariant with the level of inventory. Furthermore, most models use profit maximization or cost minimization as a decision criterion to evaluate inventory decisions, ignoring the effect of the risk of holding inventories on the value of the firm.

The purpose of this chapter is threefold: (1) to develop a model that gives an explicit expression for risk of investment in inventories in a "Newsboy" type of model, (2) to derive the optimal level of investment in inventories for a value maximizing firm, taking into consideration the effect of the risk of holding inventories on the value of the firm, and (3) to provide empirical evidence on the association between inventories and risk of the firm.

Section 2.2 looks at the "Newsboy" kind of stochastic inventory model and shows that the cash flows from holding inventories are equivalent to the cash flows from a portfolio that consists of a long position in risky bonds of a hypothetical firm and a short position in risk-free, pure discount bonds. The Sharpe (1964)-Lintner (1965) capital asset pricing model (CAPM) and the Black-Scholes (1973) option pricing model (OPM) are used to develop expressions for the risk of holding inventory and the optimal level of investment in inventories. Section 2.3 discusses the implications of the model on justification procedures for investments that change setup costs, production capacity and lead time. Section 2.4 discusses the results from the empirical study on the association between inventories and the risk of the firm.

2.2. The Valuation Model

Consider a simplified model of a firm facing the following "Newsboy" kind of inventory problem. For ease of exposition the firm is referred to as Firm A. Firm A buys and sells a single product. Let C be the purchase price per unit and P be the selling price per unit. The demand, $\tilde{D}$, for the product is stochastic. The firm exists for a single discrete period of length T. At the beginning of the period Firm A decides on the inventory level, I, needed to meet demand, which is revealed at the end of the period. If demand is greater than inventory, all inventory is sold and a stockout occurs. I assume that stockouts do not involve any cash outflow. If demand is less than inventory, the firm is left with unsold stock. A holding cost of h per unit is incurred on all unsold stock. Holding costs involve a cash outflow. All unsold unit can be sold at the original purchase price C. Also assume that all cash inflows and outflows occur at the end of the period. The firm liquidates itself at the end of the period so that there are no intertemporal dynamic links in inventory planning. [1] The objective of Firm A is to choose the inventory level that maximizes its market value. [2]

As of the beginning of the period, the end-of-period cash flows of Firm A for a given inventory level, I, are uncertain, and are as follows: If the end-of-period demand is greater than or equal to the inventory, I, the cash inflow of the firm is the fixed amount $(P - C)I$, and the holding cost is zero. If the demand is less than inventory then the cash inflow is $(P - C)$ times demand, and the holding cost is h times the difference between the inventory and demand. These cash flows are shown in Figure 2.1, and consist of the uncertain cash inflow from meeting demand and the uncertain cash outflow because of inventory holding cost.

The cash flows of Figure 2.1 are equivalent to the following two cash flow streams: (1) a certain cash outflow hI, and (2) an uncertain cash inflow that is $(P - C + h)$ times demand or inventory whichever is less. These cash flow streams are illustrated in Figure 2.2. For a given inventory level, I, the cash flows in Figures 2.1 and 2.2 are identical for any end-of-period demand outcome. Therefore, the cash flows of Firm A can be visualized as those shown in Figure 2.2. The value of Firm A is the value of the uncertain cash inflow less the value of the certain cash outflow.

[1] The firm can be visualized as operating in an environment where inventory is obtained on credit and the supplier agrees to take back unsold units at the end of the period. The firm agrees to pay the holding cost on the unsold units. Holding costs may include storage costs, insurance costs, and interest charges paid to the supplier for unsold units. Other assumptions, such as payment to the supplier at the beginning of the period or unsold units have no salvage value, can easily be incorporated in the model.

[2] I am assuming that the firm is organized as an open corporation and the claims on cash flows are traded and valued in perfect capital markets. Various authors have postulated value maximization as the appropriate investment criteria for a firm under uncertainty as well as certainty. See, for example, the papers by Modigliani and Miller (1958), Lintner (1965) and Mossin (1966). Also see Fama and Miller (1972) for a rigorous treatment of the value maximization rule.

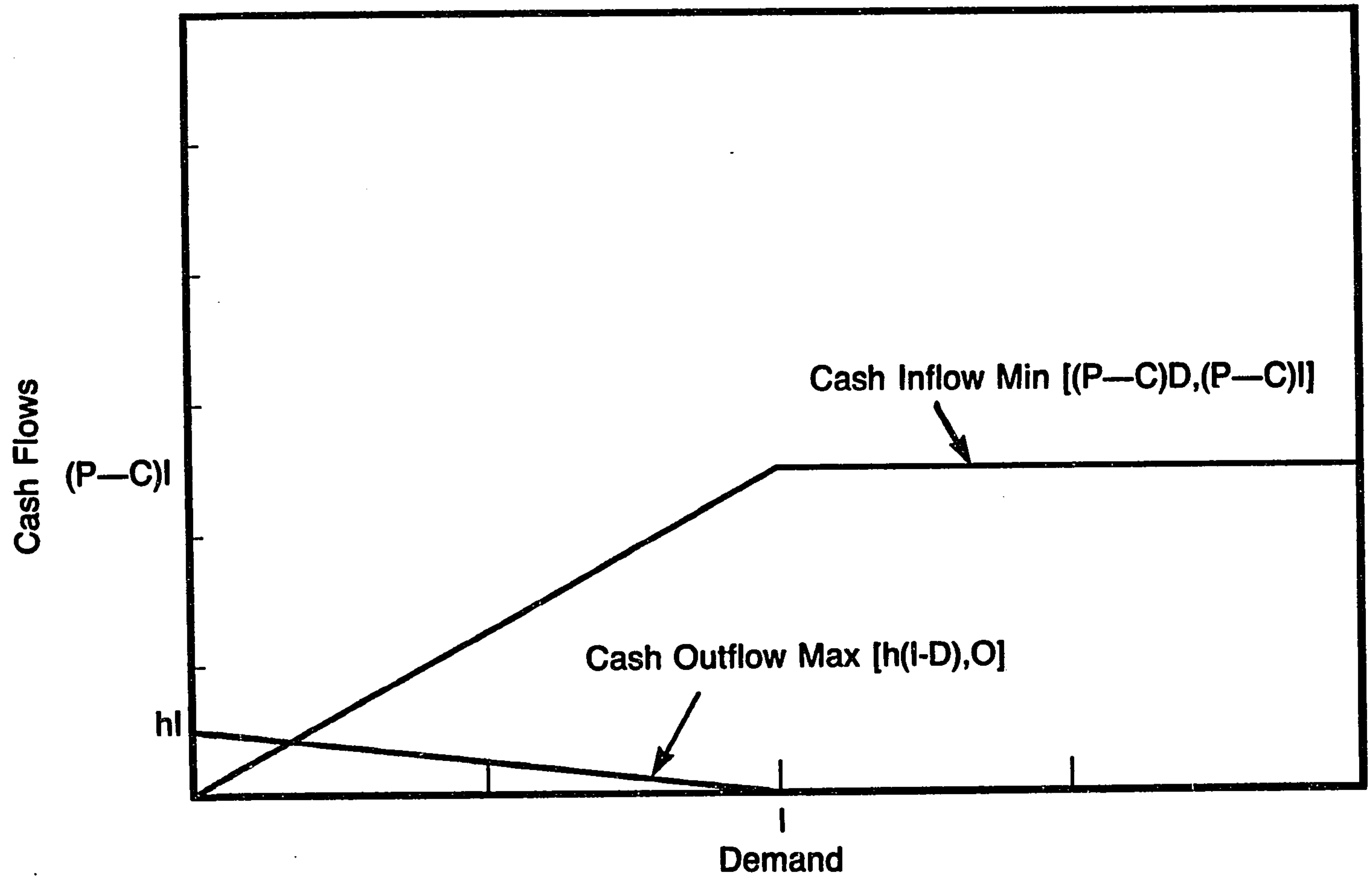

Figure 2.1. Cash flows of Firm A as a function of the end-of-period demand when inventory is I, (P—C) is the cash inflow per unit of demand met from inventory, and h is the cash outflow per unit of inventory left unsold at the end of the period.

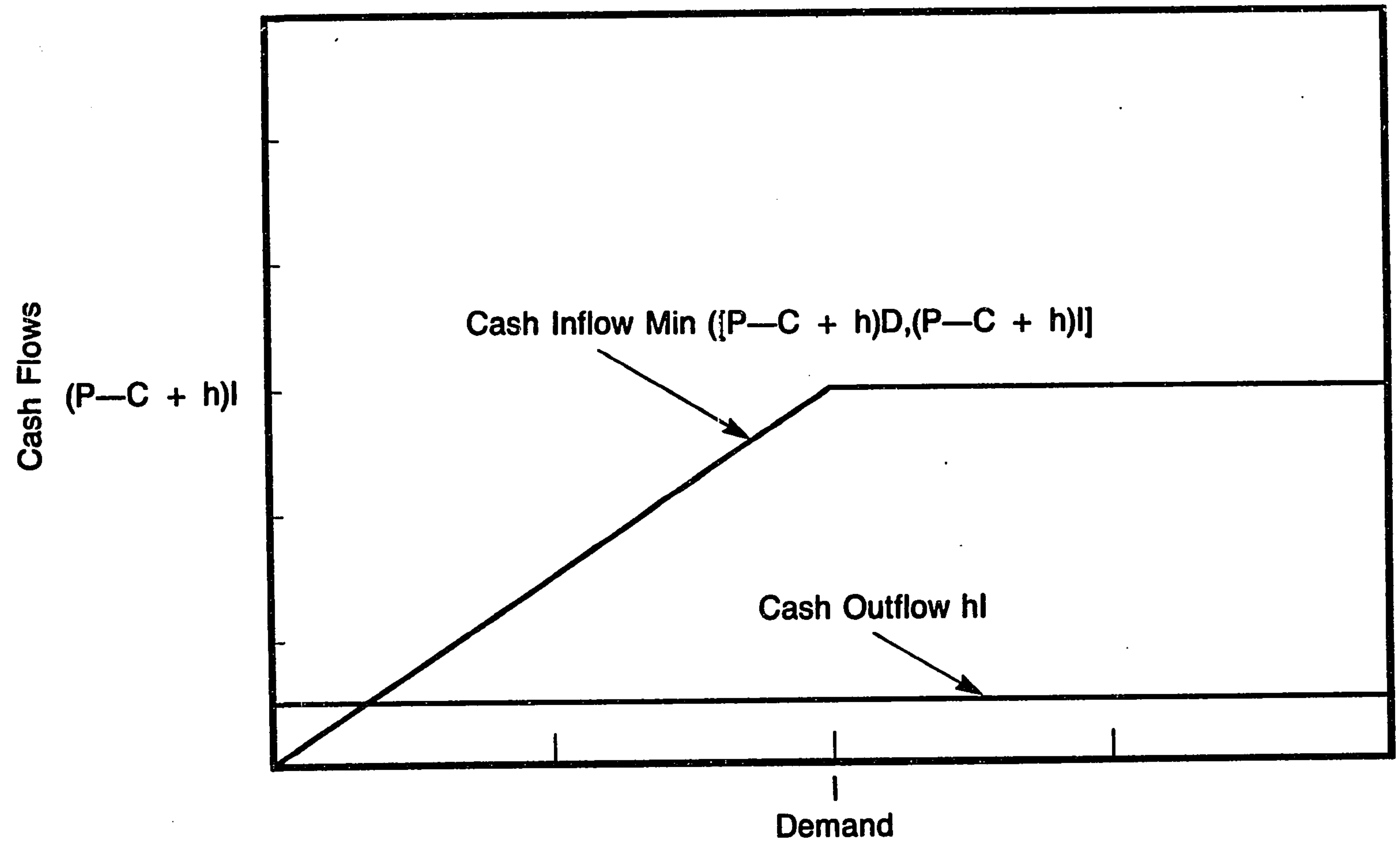

Figure 2.2. Cash flows of Firm A in Figure 2.1 can be replicated by the fixed cash outflow hI, and the cash inflow Min [(P—C + h)D,(P—C + h)I].

Now consider the inventory problem of Firm B, which sells a single product, the demand for which is identical to that of Firm A. Firm B differs from Firm A in two respects: (1) Firm B has a net cash inflow of $(P - C + h)$ per unit of demand met from inventory, and (2) the holding cost for Firm B is zero. The value maximizing inventory level for Firm B is to hold inventory large enough to meet all possible demand. The uncertain cash flows of Firm B are shown in Figure 2.3.

Suppose at the beginning of the period Firm B issues bonds of face value $(P - C + h)I$, the proceeds from which are paid out to the shareholders of the firm in the form of dividends. The bonds give the bondholders first claim on the end-of-period cash flows of Firm B, and bondholders have priority over other claimants on the cash flows of the firm. The payoffs to the bondholders of Firm B are uncertain at the beginning of the period and depend on the face value of bonds and the demand. The payoffs are as follows: To the extent that the actual cash flows of the Firm B are less than or equal to $(P - C + h)I$, the bondholders receive all the cash flows and the other claimants get nothing. On the other hand, if the cash flows are greater than $(P - C + h)I$ then the bondholders receive the fixed amount $(P - C + h)I$, which equals the face value of the bonds. The payoffs to the bondholders are shown in Figure 2.4.

Now suppose Firm A chooses inventory level I and Firm B issues bonds with a face value of $(P - C + h)I$. Consider a portfolio that consists of a long position in the bonds of Firm B, and a short position in risk-free, pure discount bonds with a face value of hI. Assume that both the risky and the pure discount bonds mature immediately after the demand is revealed. It is easy to see that the end-of-period cash flows from holding this portfolio are the same as the cash flows of

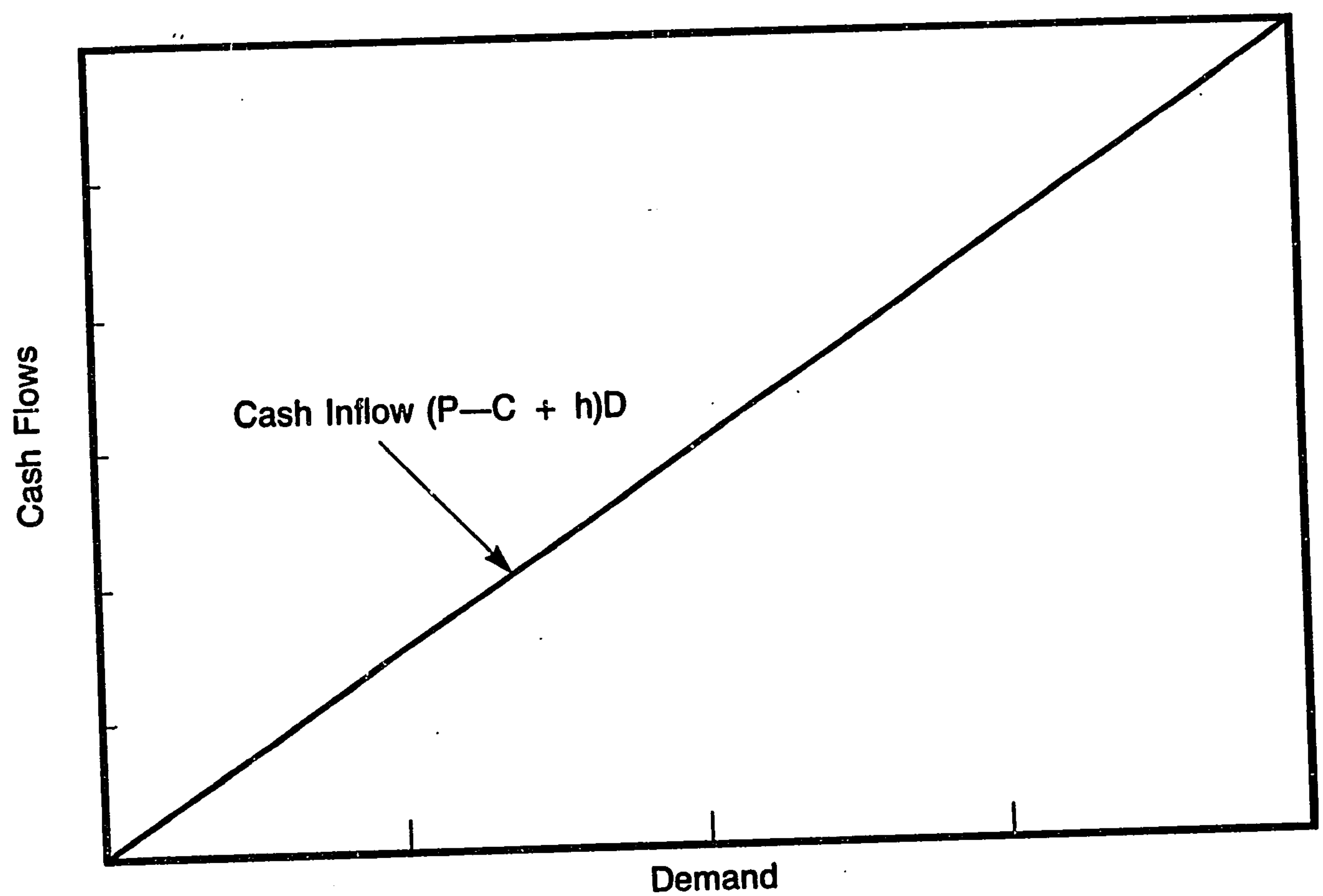

Figure 2.3. Cash flows of Firm B as a function of the end-of-period demand when (P—C +h) is the cash inflow per unit of demand.

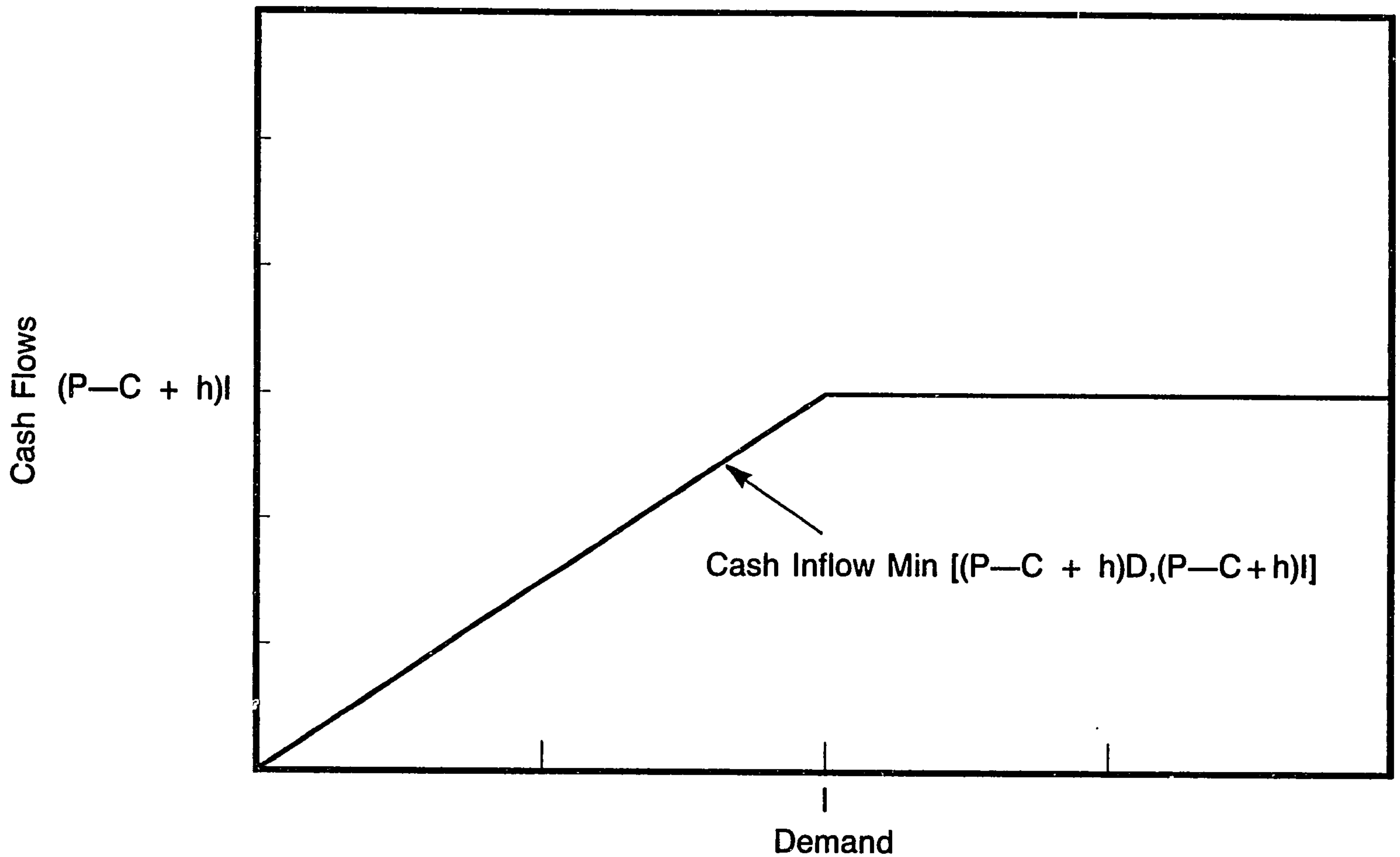

Figure 2.4. Cash flows to the bondholders of Firm B, with bonds of face value (P—C+h)I, as a function of the end-of-period demand.

Firm A, shown in Figure 2.2. Therefore, the value of Firm A must be the same as the value of the portfolio. Note that the value of the portfolio is the difference between the value of the bonds of Firm B and the value of the pure discount bond. The objective of Firm A is to choose the inventory level, I, that maximizes the value of the firm. This is equivalent to choosing $(P - C + h)I$, the face value of the bonds of Firm B, and hI, the face value of the pure discount bonds, so that the the value of the portfolio is maximized. [3]

The value of the pure discount bonds is the face value of the bonds discounted at the riskless rate of return. The valuation of bonds has been extensively studied in the financial economics literature using the Black-Scholes (1973) option pricing model (OPM). After the publication of their paper in which the pricing models for simple put and call options were originally derived, much work has been done using their option pricing model to analyze the pricing of corporate liabilities. [4] Black and Scholes suggest that option pricing model can be used to price the debt and equity of a levered firm. Black-Scholes call option pricing model provides the correct valuation of the equity and debt of the firm under the following assumptions: (1) The firm issues bonds which prohibit dividend payments until after the bonds are paid off. The bonds mature at time T and bondholders are paid first (if possible) and the residual is paid to the stockholders. (2) The value of the firm is unaffected by the issue of bonds. (3) The value of the firm is lognormally distributed with constant variance rate of return. (4) There is a known constant riskless rate.

[3] Under slightly different assumptions for the cash flows of Firm B, it can be shown that the cash flows of Firm A can be replicated by the cash flows from buying bonds and selling a put option. A put is an option to sell a share of the firm at the maturity date of the contract for a stated amount, the exercise price.

[4] For an excellent review of this literature see Smith (1976, 1979).

Issuing bonds is equivalent to the stockholders selling the firm to the bondholders for the proceed of the issue plus a call option to repurchase the firm from the bondholders with an exercise price equal to the face value of the bonds. Applying the Black-Scholes call option solution yields

$$E = VN(a_1) - e^{-r_F T} FN(a_2), \tag{2.1}$$

where E is the value of the equity, V is the value of the firm, σ^2 is the instantaneous variance rate on V, F is the face value of the bonds of the firm, T is the maturity date of the bonds, r_F is the instantaneous riskless rate, $N(.)$ is the standarized normal cumulative probability density function, and

$$a_1 = \frac{\ln(V/F) + (r_F + \sigma^2/2)T}{\sigma\sqrt{T}},$$

and

$$a_2 = a_1 - \sigma\sqrt{T}.$$

The value of bonds, D, is then

$$\begin{aligned} D &= V - E \\ &= VN(-a_1) + e^{-r_F T} FN(a_2). \end{aligned} \tag{2.2}$$

Using equation (2.2) to value the bonds of Firm B in our inventory model, the value of Firm A, V_A, as a function of its inventory level, I, is

$$V_A = V_B N(-a_1) + e^{-r_F T}(P - C + h)IN(a_2) - e^{-r_F T} hI, \tag{2.3}$$

where V_B is the value of Firm B, σ_B^2 is the instantaneous variance rate on V_B, $(P - C + h)$ is the face value of the bonds of Firm B, T is the length of the discrete period for which inventory is held, and

$$a_1 = \frac{ln(V_B/(P - C + h)I) + (r_F + \sigma_B^2/2)T}{\sigma_B\sqrt{T}},$$

and

$$a_2 = a_1 - \sigma_B\sqrt{T}.$$

In equation (2.3), the sum of the first two terms is the present value of the bonds of Firm B with face value of $(P - C + h)I$. The final term is the value of the pure discount bonds with face value of hI.

To use equation (2.3) to determine the inventory level that maximizes the value of Firm A, we need to know the value of Firm B and the instantaneous variance rate of Firm B. Recall that Firm B has the same demand distribution as Firm A. Furthermore, Firm B has a net cash inflow of $(P - C + h)$ per unit of demand met from inventory and has zero holding costs. Assume that the demand, $\tilde{D}$, is lognormally distributed with mean $\bar{D}$ and instantaneous variance of σ_D^2. [5] Since the demand is revealed at the end of a discrete period of length T, the variance of demand over the period is $\sigma_D^2 T$. The uncertain cash flow of Firm B, $\tilde{X_B}$, can be written as

$$\tilde{X_B} = (P - C + h)\tilde{D}. \tag{2.4}$$

We use the capital asset pricing model (CAPM) to determine the value of Firm B, V_B. [6] According to the CAPM the equilibrium value of the firm can be written as:

$$V_B = \frac{E(\tilde{X_B}) - \lambda \mathrm{Cov}(\tilde{X_B}, \tilde{R}_m)}{1 + R_F}, \tag{2.5}$$

where R_F is the risk-free rate of return over a discrete period of length T; [7] $\tilde{R}_m$ is

[5] The assumption that demand is lognormally distributed ensures that the value of Firm B is also lognormally distributed. This assumption is necessary to use the Black-Scholes call option pricing model for valuing bonds.

[6] The derivation of the CAPM in a discrete time framework can be found in Sharpe (1964) and Lintner (1965), and in a continuous time framework in Merton (1973).

[7] R_F is the instantaneous risk-free rate of return r_F, continuously compounded over period T.

the discrete period rate of return on the portfolio that consists of all risky assets in the market, and has an expected value of $E(\tilde{R}_m)$ and a standard deviation of σ_m;[8] $\lambda = [E(\tilde{R_m}) - R_F]/\sigma_m^2$ is the market price per unit of risk; $\text{Cov}(\tilde{X_B}, \tilde{R}_m)$ is the covariance between the cash flows of Firm B and the market return; and $E(\tilde{X_B})$ is the expected value of the cash flow of Firm B. It is implicit in the CAPM that the relevant measure of risk in pricing an asset in capital markets is the nondiversifiable risk of the asset. Investors in capital markets hold well diversified portfolios and can diversify away part of the total risk of an asset by portfolio formation. Diversification cannot eliminate the risks due to the variations in the general level of the market. The covariance of the cash flows of Firm B with the market return is the measure of the nondiversifiable risk of Firm B in the market and is relevant for pricing the cash flows of Firm B.[9]

Using equation (2.5), the value of Firm B can be expressed as:

$$V_B = \frac{(P - C + h)(\bar{D} - \lambda \text{Cov}(\tilde{D}, \tilde{R}_m))}{(1 + R_F)}, \tag{2.6}$$

[8] $\tilde{R}_m$ is the instantaneous market rate of return r_m, continuously compounded over period T.

[9] Another way of using the CAPM to calculate the value of a risky cash flow is to discount the expected cash flow, $E(\tilde{X})$, by the risk-adjusted discount rate, $E(\tilde{R})$, so that

$$V = \frac{E(\tilde{X})}{1 + E(\tilde{R})}. \tag{2.5a}$$

The discount rate is given by the CAPM's general equilibrium relationship between the risk and return of an asset in capital markets. The form of this equilibrium relationship is

$$E(\tilde{R}) = R_F + \beta(E(\tilde{R}_m) - R_F), \tag{2.5b}$$

where $\tilde{R}$ is the random rate of return of an asset and $\beta = \frac{\text{Cov}(\tilde{R}, \tilde{R}_m)}{\sigma_m^2}$, is the measure of the nondiversifiable risk of an asset. If we know the beta, β, of the cash flow, using (2.5b) $E(\tilde{R})$ can be computed. See Brealey and Myers (1981, pp 183-184) for a proof on the equivalence of (2.5) and (2.5a).

where $\mathrm{Cov}(\tilde{D}, \tilde{R}_m)$ is the covariance of demand with the market return over the discrete period of length T. The uncertain rate of return on V_B over the discrete time period T is

$$\tilde{R_B} = \frac{\tilde{X_B}}{V_B} = \frac{(1+R_F)\tilde{D}}{(\bar{D} - \lambda \mathrm{Cov}(\tilde{D}, \tilde{R}_m))}. \tag{2.7}$$

The variance of the rate of return on V_B over the time period T is

$$\sigma_B^2 T = \frac{\sigma_D^2 T (1+R_F)^2}{(\bar{D} - \lambda \mathrm{Cov}(\tilde{D}, \tilde{R}_m))^2}, \tag{2.8}$$

where σ_B^2 is the instantaneous variance rate on V_B. From (2.8) we have

$$\sigma_B^2 = \frac{\sigma_D^2 (1+R_F)^2}{(\bar{D} - \lambda \mathrm{Cov}(\tilde{D}, \tilde{R}_m))^2}. \tag{2.9}$$

Substituting for V_B and σ_B^2 in (2.3), the value of Firm A can be expressed in terms of known parameters.

Next I derive an expression for the value maximizing inventory level of Firm A, and show that the value of the firm is a concave function of the inventory level. Let D_B be the value of the bonds of Firm B when Firm A's inventory level is I units. The value of Firm A can be written as

$$V_A = D_B - e^{-r_F T} h I. \tag{2.10}$$

The first partial derivative of V_A with respect to I is

$$\frac{\partial V_A}{\partial I} = \frac{\partial D_B}{\partial I} - e^{-r_F T} h. \tag{2.11}$$

The partial derivative of D_B with respect to I can be written as

$$\frac{\partial D_B}{\partial I} = \left[\frac{\partial D_B}{\partial (P - C + h) I}\right] \left[\frac{\partial (P - C + h) I}{\partial I}\right], \tag{2.12}$$

where $(P - C + h)I$ is the face value of the bonds of Firm B when Firm A's inventory level is I units. Galai and Masulis (1976) show that the partial derivative of the value of bonds, D, with respect to the face value of the bonds, F, is

$$\frac{\partial D}{\partial F} = e^{-r_F T} N(a_2) > 0. \tag{2.13}$$

Using equation (2.13), $\frac{\partial D_B}{\partial I}$ can be written as

$$\frac{\partial D_B}{\partial I} = e^{-r_F T}(P - C + h)N(a_2). \tag{2.14}$$

Substituting for $\frac{\partial D_B}{\partial I}$ in (2.11) yields

$$\frac{\partial V_A}{\partial I} = e^{-r_F T}(P - C + h)N(a_2) - e^{-r_F T}h. \tag{2.15}$$

Setting the left hand side of (2.15) to zero gives the first-order optimality condition as

$$\frac{h}{(P - C + h)} = N(a_2). \tag{2.16}$$

Multiplying both sides of (2.16) by -1 and adding 1 to each side we get

$$\frac{(P - C)}{(P - C + h)} = N(-a_2), \tag{2.17}$$

where a_2 can be simplified and written as

$$a_2 = \frac{\ln((\bar{D} - \lambda \mathrm{Cov}(\tilde{D}, \tilde{R}_m))/e^{r_F T} I) + (r_F - \sigma_B^2/2)T}{\sigma_B \sqrt{T}}.$$

The structure of the above optimality condition is similar to the condition derived in models that use expected profit maximizing as the decision criterion (see, for example, Peterson and Silver (1979)). Observe that the optimal inventory level depends, among other things, on the risk of the demand as measured by its co-variability with the market return. Since the term $\frac{(P-C)}{(P-C+h)}$ is a constant, and the optimality condition relates this term to the normal distribution, there exists a K such that

$$\frac{(P-C)}{(P-C+h)} = N(K). \quad (2.18)$$

Comparing (2.17) and (2.18) we have

$$K = -a_2 = -\frac{\ln((\bar{D} - \lambda \mathrm{Cov}(\tilde{D}, \tilde{R}_m))/e^{r_F T} I) + (r_F - \sigma_B^2/2)T}{\sigma_B \sqrt{T}}. \quad (2.19)$$

On simplifying (2.19) the optimal inventory level, I^*, can be expressed as

$$I^* = \frac{(\bar{D} - \lambda \mathrm{Cov}(\tilde{D}, \tilde{R}_m))}{e^{r_F T}} e^{(K\sigma_B\sqrt{T} + (r_F - \sigma_B^2/2)T)}. \quad (2.20)$$

It can be easily shown that the second derivative of V_A with respect to I is

$$\frac{\partial^2 V_A}{\partial I^2} = \frac{-e^{-r_F T}(P-C+h)f(a_2)}{\sigma_B \sqrt{T} I}. \quad (2.21)$$

where $f(.)$ is the standard normal density function. The right hand side of (2.21) is less than zero everywhere. Therefore, the value of Firm A is a concave function of the inventory level and will have a unique optimum.

Next consider the effect of inventory on the risk of Firm A as measured by its beta, β_A. [10] Since the cash flows of Firm A are equivalent to the cash flows from the portfolio that consists of a long position in risky bonds of Firm B and a short position in riskless bonds, the beta of Firm A equals the beta of the portfolio. The beta of the portfolio is a weighted average of the betas of the risky bonds of Firm B and the riskless bonds. The beta of the bonds of Firm B, β_D, is weighted by the ratio of the value of its bonds, D_B to the the value of the portfolio, V_A. [11] The beta of the riskless bonds is weighted by the ratio of $-e^{-r_F T} hI$ to the value

[10] See footnote 9 for a definition of beta.

[11] Note that the value of the portfolio equals the value of Firm A, V_A. The value of the bonds of Firm B equals $(V_A + e^{-r_F T} hI)$.

of the portfolio. Since the beta of the riskless bonds equals zero, the beta of Firm A, β_A, can be written as

$$\beta_A = \frac{\beta_D D_B}{V_A} = \frac{\beta_D (V_A + e^{-r_F T} hI)}{V_A}. \tag{2.22}$$

In words, the beta of Firm A for a given inventory level I is the product of Firm B's beta of bonds with a face value of $(P - C + h)I$ and the ratio of the value of Firm B's bonds to the value of Firm A.

The face value of bonds of Firm B increases with an increase in the inventory level of Firm A. Galai and Masulis (1976) show that the beta of bonds is an increasing function of the face value of bonds. Using their result it can be seen that the beta of the bonds of Firm B increases with an increase in the inventory level of Firm A. Furthermore, the ratio of the value of bonds of Firm B to the value of Firm A is an increasing function of the inventory level of Firm A. [12] Therefore, with an increase in the inventory level, the beta of Firm A increases, that is, the risk of Firm A increases with its inventory level.

A more intuitive measure of risk is the discount rate or the opportunity cost of capital. From CAPM's equilibrium relation between the risk and return of an asset in capital markets, the risk of Firm A in terms of discount rate can be expressed as

$$E(\tilde{R_A}) = R_F + \beta_A(E(\tilde{R_m}) - R_F), \tag{2.23}$$

where $E(\tilde{R_A})$ is the expected discount rate of Firm A.

[12] The partial derivative of the ratio of the value of bonds of Firm B to the value of Firm A, $\frac{(V_A + e^{-r_F T} hI)}{V_A}$, with respect to I can be written as $\frac{e^{-r_F T} h}{{V_A}^2}(V_A - \frac{\partial V_A}{\partial I} I)$. Using equations (2.3) and (2.15) it can be shown that $(V_A - \frac{\partial V_A}{\partial I} I)$ equals $V_B N(-a_1)$. Since $V_B N(-a_1)$ is always greater than zero, the ratio of the value of bonds of Firm B to the value of Firm A is an increasing function of the inventory level of Firm A.

For hypothetical lognormal distributions of demand and market return, Table 2.1 contains numerical values of the expected net cash flow, $E(\tilde{X})$, the value of the firm, $V(\tilde{X})$, the risk of the firm as measured by the discount rate, R, and the probability of no stockout, P, for various levels of inventory. The numerical values have been computed for different values of covariance of demand with market return. [13] The value under the expected net cash flow column is the value of the firm when the covariance of demand with the market is zero. Figures 2.5 and 2.6 depict the behavior of the value of the firm and the discount rate as a function of the inventory level for different values of the covariance of demand with the market return. The expected net cash flow curve in Figure 2.5 is the value of the firm when the covariance of demand with the market is zero. From Table 2.1, and Figures 2.5 and 2.6 we observe the following:

(1). The value maximizing inventory level is less than the inventory level

[13] In parametrizing the example, the expected market return $E(\tilde{R}_m)$, and the riskless rate of return, R_F, are estimated using the average of the time series of realized market return and riskless rate of return over the years 1975-1984 (see Ibbotson and Sinquefield (1985)). The market price per unit of risk, λ, is estimated using the average realized risk premium on the market, $(E(\tilde{R}_m) - R_F)$, and the variance of the market return, σ_m^2, over the same time period. The reason for using the time series over the years 1975-1984 is that this time period reflects recent history and is also the sample period used in the empirical study on the association between inventories and risk of the firm (see section 2.4). The estimates of the market parameters are not invariant to the length of the estimation period. Hence, if a time period other than the years 1975-1984 is used to estimate these parameters, the numerical values in the example will be different. However, as long as the expected risk premium on the market is positive, the qualitative conclusions from the example will not change. Since realized rates of return on the market can be negative, it is certainly possible that for a particular time period the market risk premium could be negative. However, from prior knowledge, the market risk premium must be positive because investors demand a risk premium for holding risky assets. Hence, a negative value of the market risk premium must be a biased-low estimate of the market risk premium. Merton (1980) suggests that models for estimating the market risk premium must include the condition that the market risk premium is positive. More sophisticated models for estimating the market risk premium are discussed in his paper.

Table 2.1

Inventory, Probability of Product Availability P, Expected Net Cash Flow $E(\tilde{X})$, the Value of the Firm $V(\tilde{X})$, and the Discount Rate R in Percentage, for Various Values of the Covariance of Demand with the Market Return.

Inventory	P	$E(\tilde{X})$	$COV(\tilde{D},\tilde{R}m)=5000$		$COV(\tilde{D},\tilde{R}m)=7500$		$COV(\tilde{D},\tilde{R}m)=10000$	
			$V(\tilde{X})$	R%	$V(\tilde{X})$	R%	$V(\tilde{X})$	R%
204000	0.58	1,603	1,498	15.8	1,439	20.6	1,377	26.2
212000	0.64	1,622	1,509	16.4	1,447	21.4	1,381	27.3
220000	0.69	1,637	1,516	16.9	1,451	22.1	1,383	28.3
228000	0.74	1,648	1,520	17.4	1,452	22.9	1,382	29.2
236000	0.78	1,655	1,522	17.8	1,450	23.5	1,378	30.1
248000	0.83	1,660	1,519	18.5	1,445	24.4	1,370	31.3
264000	0.88	1,657	1,508	19.2	1,432	25.5	1,354	32.6
272000	0.90	1,652	1,501	19.5	1,423	25.9	1,345	33.3
304000	0.96	1,619	1,460	20.5	1,380	27.5	1,299	35.4
400000	1.00	1,462	1,298	22.7	1,215	31.1	1,133	40.7

The numerical results have been computed using the following parameters:

Demand is lognormally distributed with mean of 200000 units per year, and standard deviation of 50000 units. Selling price is \$20/unit, purchase cost is \$10/unit, and holding cost is \$2/unit.

The following parameters are estimated using data over the years 1975-1984 from Ibbotson and Sinquefield (1985): (1) Expected return on a market portfolio of assets, $E(\tilde{R}_m)$, is 15.7%/year, (2) standard deviation of the returns on a market portfolio of assets, σ_m, is 15.0%/year, (3) risk-free rate of return, R_F, is 8.9%/year, and (4) market price per unit of risk, λ, is 3.0 percent per unit of variance.

Expected net cash flow $E(\tilde{X})$, and value of the firm $V(\tilde{X})$, figures are in thousands of dollars.

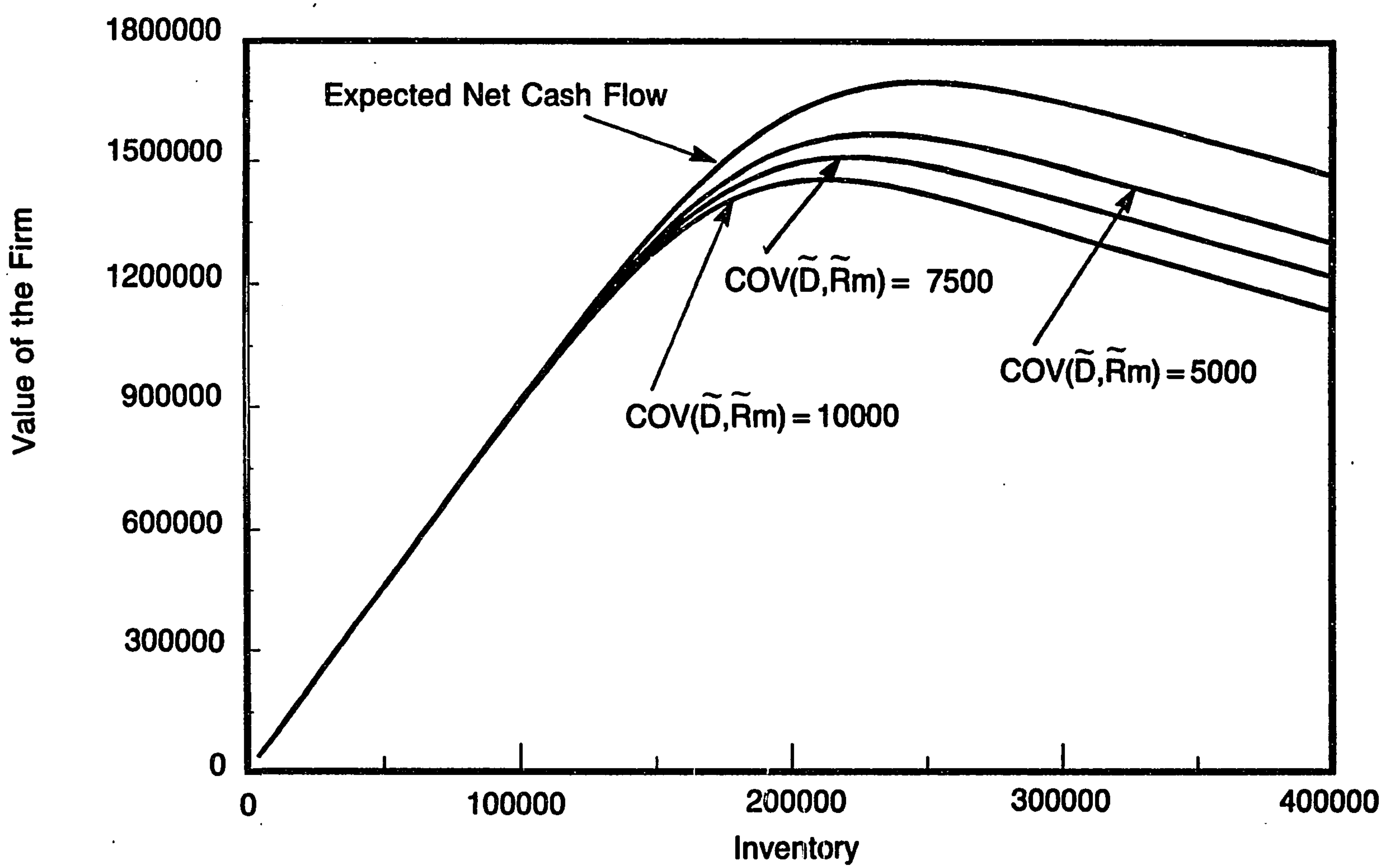

Figure 2.5. Value of the firm as a function of inventory for various values of the covariance of the demand with the market return. The expected net cash flow curve is the value of the firm when the covariance is zero.

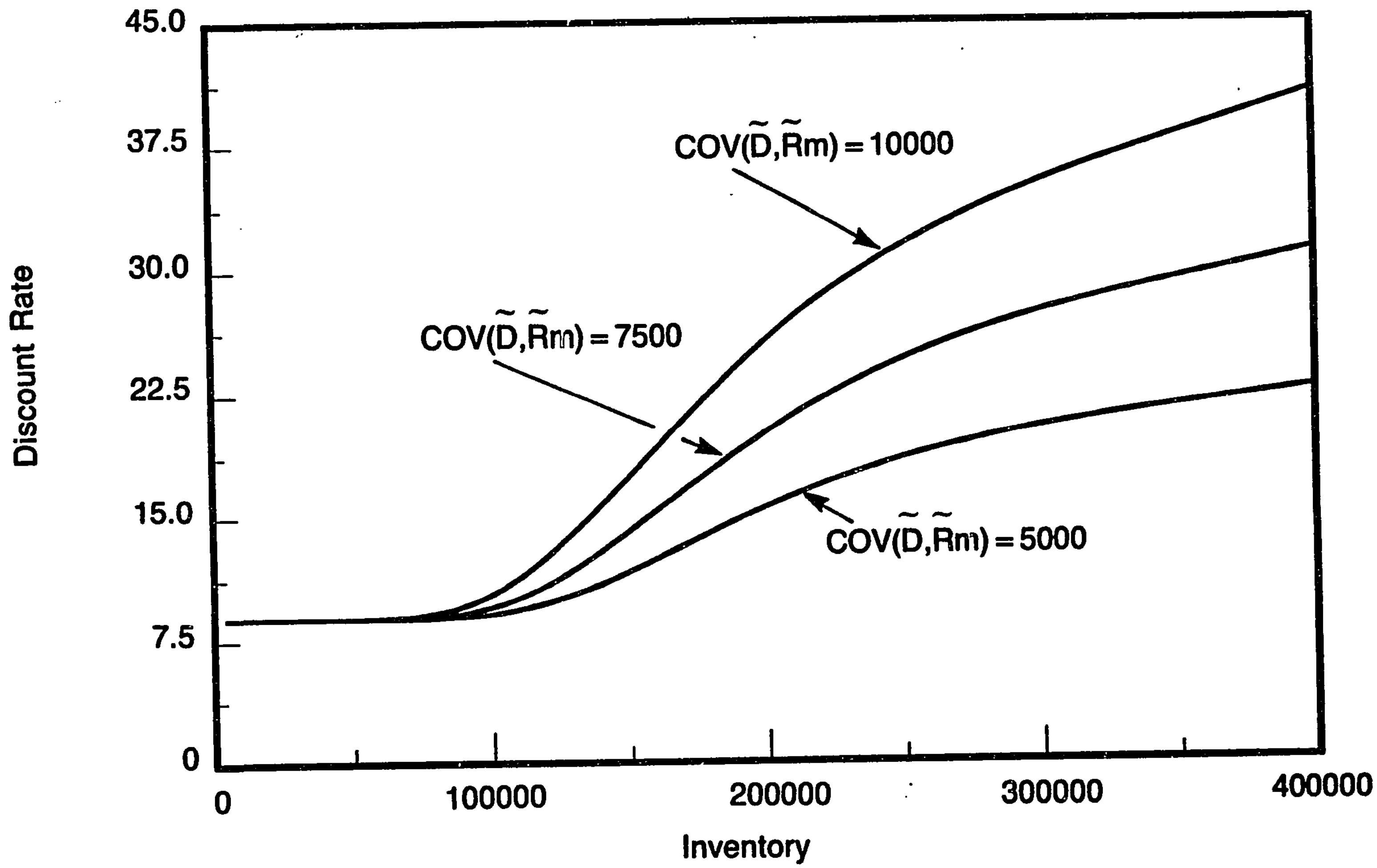

Figure 2.6. Discount rate (in percent) of the cash flows as a function of inventory for various values of the covariance of the demand with the market return.

that maximizes the expected net cash flow. The difference between the two inventory levels increases with an increase in the covariance of demand with the market return. In Table 2.1, the difference between the expected net cash flow maximizing and the value maximizing inventory levels varies from 4.8% to 11.3%. The probability of product availability varies from a high of 0.83 to a low of 0.69. Furthermore, when the inventory is held constant, an increase in the covariability of the demand with the market return decreases the value of the firm.

(2). An increase in the covariance of the demand with the market return decreases the optimal level of inventory. Ceteris paribus, an increase in the covariance increases the riskiness of the demand, which increases the opportunity cost of capital of investment in inventory. Therefore, the optimal level of inventory decreases.

(3) When the covariance of demand with the market return is held constant, an increase in the inventory increases the discount rate. Also, when the inventory is held constant, an increase in the covariance of the demand with the market return increases the discount rate.

The valuation model developed above differs from the traditional models in management science literature in three ways. First, it focuses on the cash flow implications of the inventory decision and the effect they have on the value of the firm. Traditional models are developed around the physical flows of the system. More often than not, the cash flow implications of the models do not correspond to the physical flows of the system. [14] Secondly, traditional models include an

[14] The papers by Beranek (1967), Trippi and Levin (1974), Grubbstrom (1980), Grubbstrom and Thorstenson (1981), Gurnani (1983) and Rummel (1985) have examined the cash flow aspects of inventory decisions by modelling these decisions in present value terms. These papers analyze inventory decisions under certainty where the cash flows are certain and the appropriate discount rate is the risk-free rate of return.

opportunity cost of capital which is assumed to be independent of the inventory level. This assumption is valid only in deterministic inventory models where the cash flows are certain, and the correct opportunity cost of capital is the risk-free interest rate. Under uncertainty, the risk of the cash flows, and hence the opportunity cost of capital, is an increasing function of the inventory level. Therefore, it is incorrect to assume a constant opportunity cost of capital in stochastic inventory models. Finally, our model considers the effect of risk on the value of the firm by discounting the risky cash flows at the market price of risk. Traditional models use expected profit maximization or expected cost minimization as the decision criterion, ignoring the effect of risk.

2.3. Implications for Capital Budgeting Procedures

The results derived above have implications for capital budgeting procedures for investments that alter the parameters of the production-distribution system of the firm (such as setup costs, lead time, or production rate). The implications are illustrated with a numerical example that compares a production-distribution system where the holding costs are positive with a system where the holding costs are zero. Holding costs are zero in the sense that the production-distribution parameters are such that the firm need not hold any inventories. This will be the case when setup costs or lead times are zero or the production rate is infinite. This assumption is made to illustrate the revenue and the risk implications of altering the system parameters. [15]

Table 2.2 presents numerical results on the optimal inventory, the value of the

[15] The concepts of Just-In-Time manufacturing and Kanban developed and practised by Japanese are examples where setup costs and inventory holding costs are minimal. New technologies like Flexible Manufacturing Systems and Robotics have the flexibility to set the machine for a new part or product by just loading a program so that the setup costs are very small.

firm, the probability of product availability, and the discount rate when holding costs are zero. It also gives the corresponding results from Table 2.1 where the holding costs are positive. Column 10 gives the risk-adjusted total holding costs, which is relevant only when holding costs are positive. [16] The last column of Table 2.2 gives the difference between the value of optimal inventory policies with zero holding cost and positive holding costs.

The value of reducing holding cost to zero is given in column 11. This value can be broken into three components: (1) the value from the direct savings in holding costs, (2) the value from the higher probability of product availability, and (3) the value from lower risks.

The value of savings in inventory holding costs is the risk-adjusted holding cost under column 10. Observe that this value is less than the value in column 11. Also note that with positive holding costs the optimal inventory policy is such that the probability of product availability is less than one, whereas when the holding costs are zero the probability of product availability is one. An increase in the probability of product availability increases the expected revenues of the firm, thereby increasing the value of the firm. Finally, from Tables 2.1 and 2.2 we find that the discount rate when holding costs are positive and the probability of product availability is one, is greater than when holding costs are zero and the probability of product availability is one. This means that with zero holding costs the risk of the cash flows is reduced, which increases the value of the firm.

The revenue and risk implications of altering system parameters may be ignored in many capital budgeting procedures. Often, these procedures are oriented

[16] We can think of the firm as buying an insurance policy, whereby the insurance firm agrees to pay all holding costs. The insurance company will charge a price which equals the risk-adjusted value of the uncertain cash outflow. The price of buying this insurance policy is the risk-adjusted cost of holding inventory.

Table 2.2

Comparison of the Optimal Values of Inventory, the Probability of Product Availability, the Value of the Firm, and the Discount Rate When Holding Costs are Zero with the Values When the Holding Costs are Positive.

	Holding Cost = 0				Holding cost = 2 per unit					
$COV(\tilde{\delta},\tilde{R}m)$ (1)	Inventory (2)	Probability of Product Availability (3)	Value of the Firm (4)	Discount Rate (5)	Inventory (6)	Probability of Product Availability (7)	Value of the Firm (8)	Discount Rate (9)	Risk-Adj. Holding Costs (10)	Difference between col 4 and 8 (11)
5000	400,000	1.00	1,693	18.0	236,000	0.78	1,522	17.8	106	171
7500	400,000	1.00	1,625	23.0	228,000	0.74	1,452	22.9	105	173
10000	400,000	1.00	1,557	28.5	220,000	0.69	1,383	28.3	105	174

The numerical results have been computed using the following parameters:

Demand is lognormally distributed with mean of 200000 units per year, and standard deviation of 50000 units. Selling price is \$20/unit, purchase cost is \$10/unit, and holding cost is \$2/unit.

The following parameters are estimated using data over the years 1975-1984 from Ibbotson and Sinquefield (1985): (1) Expected return on a market portfolio of assets, $E(\tilde{R}_m)$, is 15.7%/year, (2) standard deviation of the returns on a market portfolio of assets, σ_m, is 15.0%/year, (3) risk-free rate of return, R_F, is 8.9%/year, and (4) market price per unit of risk, λ, is 3.0 percent per unit of variance.

Risk-adjusted holding costs are relevant only when holding costs are positive.

Value of the firm and risk-adjusted holding costs figures are in thousands of dollars.

towards consideration of direct cost reduction and the comparison of initial outlays against direct cost savings. In the setting of our example, the initial outlay will be compared with the savings in the holding costs. If the initial outlay is less than the savings in holding cost, the NPV of the investment opportunity is positive, and the firm will accept the investment opportunity. But if the initial outlay is more, firms can make the wrong decision. Based on direct cost savings firms will reject this investment opportunity, whereas if the other benefits are considered it could be a positive NPV project which should be accepted by the firm.

2.4. Empirical Evidence on the Association Between Inventories and Risk of the Firm.

The model developed earlier shows that the relation between the risk of the firm and its inventory is positive, that is, firms holding higher inventories are more risky. I conducted a study to test this proposition empirically. The study has two parts. The first part discusses the notion that holding inventory creates operating leverage similar to the leverage from the commitment to fixed operating costs. This notion helps develop a relation between the risk of the firm and its average inventory level, which is empirically testable using ordinary least squares estimation techniques. The second part describes the sample of firms and the principal sources of data used for the study, the methodology used to estimate the variables in the regression model, and the results from the study. The results weakly support the hypothesis that firms holding higher inventories are in fact more risky.

A firm's operating leverage is defined as the ratio of variable profits (revenue minus variable costs) to operating profits (variable profits minus fixed operating costs). Rubinstein (1973), Brenner and Schmidt (1978), and Gahlon and Gentry

(1982) show that the risk of the firm is an increasing function of the operating leverage. Lev (1974) provides empirical evidence that operating leverage is one of the determinants of the risk of the firm, and that firms with high operating leverage are more risky. However, the connection between inventories and operating leverage has not been made.

Most firms invest a significant amount of capital in cycle stocks and safety stocks. Although the level of such inventories fluctuates over time, most firms still hold some average level of inventories to support efficient utilization of their productive resources. The need for such inventories is driven both by internal factors such as set-up costs, production lead times, and production capacity, as well as from competitive factors such as better customer response times, and high probability of product availability. At least in the short-run, the average level of investment in inventories necessary to operate effectively is fixed. Firms incur holding costs in managing their investments in inventories and the total holding costs are like fixed costs. It is in this sense that holding inventory creates operating leverage similar to the operating leverage from the commitment to fixed manufacturing costs. [17]

The relation between the operating leverage from holding inventories and the

[17] Manufacturing firms in the automotive industry follow the practice of holding inventory of cars that equals 2 months of sales. Deviations from this target level of finished car inventories are corrected by increasing production if inventories are below the 2 month target, or by reducing production and providing incentives for customers to buy cars if inventories are above the 2 month target. On average, inventories in the distribution systems equals 2 months of sales. Over the years 1965-1984, the actual ratio of end-of-year inventories to annual passenger car sales by United States automobile manufacturers varied between 1.7 to 2.84 months of sales. The average and standard deviation of the ratio over the same period are 2.2 and .314 months of sales, respectively. The end-of-year inventory, and annual car sales figures are obtained from Motor Vehicle Manufacturers Association(MVMA) Motor Vehicles Facts and Figures (1985).

risk of the firm is developed under the following assumptions. Consider a firm that buys and sells a single product, which has stochastic demand. Let P be the selling price per unit, C be the purchase cost per unit, and h be the holding cost per unit. Assume that the inventory held by the firm is sufficient to meet all possible demand without any backordering. Let I be the average level of inventory held by the firm. The total inventory holding cost incurred during the period is the product of the holding cost per unit, h, and the average inventory level, I. The firm exists for a single period and all cash flows occur at the end of the period. Appendix A shows that the risk of the firm as measured by its beta, β, can be expressed as a function of the average inventory level by the following equation:

$$\beta = \frac{\beta_D}{1 - \frac{hI}{(P-C)(\bar{D}-\lambda \mathrm{Cov}(\tilde{D}, \tilde{R}_m))}}, \tag{2.24}$$

where β_D is defined as the beta of demand, and is given by:

$$\beta_D = \frac{(1 + R_F)\mathrm{Cov}(\tilde{D}, \tilde{R}_m)}{\sigma_m^2(\bar{D} - \lambda \mathrm{Cov}(\tilde{D}, \tilde{R}_m))}, \tag{2.25}$$

and R_F is the risk-free rate of return, $\bar{D}$ is the expected value of demand, λ is the market price per unit of risk, $\mathrm{Cov}(\tilde{D}, \tilde{R}_m)$ is the covariance of the demand with the market return, $\tilde{R}_m$, and σ_m^2 is the variance of the market return.

The beta of demand is the measure of the risk of demand and can be interpreted in the following way. Suppose that there exists a firm that can meet its uncertain demand without holding any inventories. Such a firm faces the risk of the demand. The beta of this firm is the beta of demand, β_D. In other words, the beta of demand is the risk of an "unlevered" firm.

In equation (2.24) the term $1/(1-\frac{hI}{(P-C)(\bar{D}-\lambda \mathrm{Cov}(\tilde{D}, \tilde{R}_m))})$, which can be written as $\frac{(P-C)(\bar{D}-\lambda \mathrm{Cov}(\tilde{D}, \tilde{R}_m))}{(P-C)(\bar{D}-\lambda \mathrm{Cov}(\tilde{D}, \tilde{R}_m))-hI}$, measures the degree of operating leverage from holding inventories. The degree of operating leverage will be greater than 1 when the following two conditions are satisfied: (1) the total inventory holding cost, hI, is less than the certainty equivalent of the total contribution, $(P - C)(\bar{D} - \lambda \mathrm{Cov}(\tilde{D}, \tilde{R}_m))$, and (2) the certainty equivalent of demand, $(\bar{D} - \lambda \mathrm{Cov}(\tilde{D}, \tilde{R}_m))$, is greater than zero. If these two conditions are not satisfied it can be shown that the value of the firm will be negative, which essentially means that the firm will not survive in the long run. Without being overly restrictive we assume that when the inventory level is positive the degree of operating leverage is greater than 1. Equation (2.24) shows that firms holding inventories are "levered", in the sense that the beta of the firm is greater than the beta of an unlevered firm, β_D. Ceteris paribus, an increase in inventory increases the degree of operating leverage of the firm which means that the beta of the firm also increases. Therefore, firms holding higher inventories are more risky.

There are two methods to test for the nonlinear multiplicative effect of inventories on the beta of the firm. First, is to use nonlinear least squares techniques. Secondly, a logarithmic transformation of equation (2.24) gives a linear equation that is testable using linear least squares. I use the second method. Taking the natural logarithm on both sides of (2.24) yields

$$\ln \beta = \ln \beta_D - \ln\left(1 - \frac{hI}{(P - C)(\bar{D} - \lambda \mathrm{Cov}(\tilde{D}, \tilde{R}_m))}\right), \qquad (2.26)$$

which is the equation used in the empirical study.

The sample of firms for the empirical study is selected from four industries in the retailing industry. These industries and their Compustat industry numbers are: Department Stores (5311), Variety Stores (5331), Grocery Stores (5411), and Drug Stores (5912). The reason for selecting the sample from the retailing industry is that the noise from the allocation of fixed operating costs to cost of goods sold and inventory, is likely to be less in retailing firms than in manufacturing firms. Data were derived both from the Standard and Poor's Compustat Tape and from the Center for Research in Security Prices Monthly Rate of Return Tape. Candidates for inclusion in the sample were required to have data over the years 1975-1984, required for this study (e.g., sales, cost of goods sold, inventory, debt-equity ratio, and monthly returns). The 10-year restriction is used to obtain a reasonable number of observations to estimate the covariance terms in the regression equation. The final sample consists of 54 firms. The breakdown of the 54 firms by industry are: 13 Department Stores, 12 Variety Stores, 19 Grocery Stores, and 10 Drug Stores.

To estimate the term $(P - C)(\bar{D} - \lambda \mathrm{Cov}(\tilde{D}, \tilde{R}_m))$ data on selling price, purchase cost, and demand are required. Since none of these data are available on the Compustat Tape, the term $(P - C)(\bar{D} - \lambda \mathrm{Cov}(\tilde{D}, \tilde{R}_m))$ is estimated as follows: Notice that $(P - C)\bar{D}$ is the expected total contribution, and $(P - C)\mathrm{Cov}(\tilde{D}, \tilde{R}_m)$ is the covariance of the total contribution with the market return. The total contribution can be measured as sales less cost of goods sold. Compustat Tape has data on sales and cost of goods sold, on an annual basis. Using these data the time-series of total contribution is generated. The average value of this time-series, and the covariance of this time-series with the return on the market portfolio are computed. For the purpose of this study the value-weighted index of all New York Stock Exchange stocks is used to measure the return to the market portfolio. The market price per unit of risk is estimated by using the time-series of the market return and the return on a portfolio of United States government Treasury Bills over the years 1975-1984 (see Ibbotson and Sinquefield (1985)). The estimate of the market price per unit of risk is 3.0 percent per unit of variance.

The fiscal year-end dollar inventory data are available on the Compustat Tape. The annual yield on new AA industrial bonds is used as the proxy for holding cost per dollar of inventory. This long-term interest rate is used because inventory is viewed as a long-term investment. The total inventory holding costs are estimated as the fiscal year-end inventory times the average yield on new AA industrial bonds during that year. For each firm in the sample, the inventory holding cost is computed for each year over the years 1975-1984, and the average over these years is used.

To estimate the beta of demand (see equation (2.25)) annual sales are used as a proxy for demand. Using the data over the years 1975-1984, the average sales, and the covariance of sales with the market return are computed. From Ibbotson and Sinquefield (1985) the average risk-free rate of return over the years 1975-1984 is estimated at 8.9%, and the standard deviation of market return, over the same period, at 15.0%. Using these estimates, the beta of demand is computed.

The beta of each firm's stock is estimated by regressing the monthly return on the firm's stock on the monthly market return during the years 1975-1984, using the following regression equation:

$$R_{jt} = \alpha_j + \beta_j R_{mt} + \tilde{u_{jt}}, \qquad (2.27)$$

where R_{jt} is the month t return on the stock of Firm j, R_{mt} is the month t return on the market portfolio, β_j is the estimate of beta of the stock of firm j, and $\tilde{u_{jt}}$ is the disturbance term for month t.

The betas are adjusted for the effect of financial leverage using a technique developed by Hamada (1972). The financial leverage of the firm affects the beta of the stock. In general, the higher the financial leverage, the higher would be the beta of the firm's stock (see Brealey and Myers 1981, pp. 169-175 for a discussion of this issue). Adjusting for financial leverage would enable us to concentrate on the effect of inventories on the beta of the firm. The betas are adjusted or unlevered using Modigliani and Miller's model (1963). Their model assumes that the value of the levered firm is equal to the value of the unlevered firm plus the value of the tax shield on interest payment on debt. The relation between the beta of the unlevered firm's stock, β_U, and the beta of the levered firm's stock, β_L, is given by

$$\beta_U = (\frac{\beta_L}{1+(1-t)D/E}), \tag{2.28}$$

where D is the market value of debt, E is the market value of the equity, and t is the firm's income tax rate. I assumed a tax rate of 46%. For unlevering the betas, debt is measured as the sum of the book values of long-term debt, preferred stock and current liabilities, and the equity as the number of shares outstanding at the end of the year times price per share at the end of year. The average of the debt-equity over the years 1975-1984 is used. [18]

I pooled the observations from the four industries and regressed the unlevered stock betas on the betas of demand and the operating leverage from holding inventories, using the following equation:

$$\ln\beta = a_1 + b_1 \ln\beta_D + c_1 \ln(1 - \frac{hI}{(P-C)(\bar{D} - \lambda \mathrm{Cov}(\tilde{D}, \tilde{R}_m))}) + \tilde{e}, \tag{2.29}$$

where $\tilde{e}$ is the disturbance term of the regression equation.

Comparing (2.26) and (2.29) we expect a_1 to be equal to zero, b_1 to be equal to 1, and c_1 to be equal to -1. The estimates of the coefficients for this regression

[18] The betas are also unlevered assuming that the value of tax-shields is zero which is consistent with Miller's model (1977). The regression results from this method of unlevering the betas are similar to the results obtained from unlevering the betas using Modigliani and Miller's model (1963).

equation are:

$$a_1 = -1.576 \text{ (standard error is 0.173)}$$

$$b_1 = 0.534 \text{ (standard error is 0.153)}$$

$$c_1 = -4.639 \text{ (standard error is 2.280)}$$

The F-value and adjusted R^2 are 11.896 and 0.291, respectively.

The regression results support the hypothesis that inventories affect the risk of the firm. The signs of the estimated coefficients are in the predicted direction. Perhaps, the most interesting result is the negative value of the estimate of the coefficient c_1. The theoretical model predicts that this coefficient is equal to -1. The estimate of this coefficient is significantly different from zero (t-value is -2.03), but is not significantly different from its hypothesized value of -1 (t-value is -1.6) at the 0.05 level. The estimate of b_1 is significantly different from both zero (t-value is 3.5) and its hypothesized value of 1 (t-value is -3.05) at the 0.05 level.

The estimate of the intercept, a_1 is significantly different from its hypothesized value of zero (t-value is -9.13). There are at least two reasons for this. First inventory is not the only variable that affects the beta of the firm. For example, fixed operating costs other than inventory holding costs affect the beta of the firm in the same way as inventory holding costs. The omitted variables will affect the intercept only if they are correlated with included variables.[19] Future research should identify these variables, and empirically test for their effect on the beta of the firm.

The second reason is that the independent variables in our regression equation

[19] See Maddala (1977, pp. 155-157) for a discussion of the effect of omitted variables on the estimates of the intercept and the coefficients of included variables.

are measured with error. If $\ln \beta_D$ is measured with error, plim $b_1 < 1$ in a simple regression. Furthermore, if the the average value of $\ln \beta_D$ is less than zero, plim $a_1 < 0$ (the average value of $\ln \beta_D$ is -0.09). Since the estimate of b_1 is less than 1 and the estimate of the intercept is negative, this may be due to the fact that $\ln \beta_D$ is measured with error.

There are at least two other sources of bias in our study. First, the study measures the average inventory level by the inventory reported at the end of the fiscal year. Depending on the industry and the fiscal year-end date, the inventory data used in this study may misestimate the average inventory level. The problem is not probably serious for Grocery Stores and Drug Stores, but could be serious for Department Stores and Variety Stores because of seasonal demand. This bias could have been avoided, to some extent, if quarterly data were available. Unfortunately, the Compustat Tape only has inventory data on an annual basis. Second, the estimates of inventory holding costs are biased downward. We have used a proxy for the opportunity cost of capital for investing in inventories, which is one of the components of inventory holding costs. Other inventory related costs such as handling, moving, and storage have not been considered. These costs cannot be estimated without access to internal (unpublished) data. These costs are probably different across industries. Because of these two sources of bias, estimates of inventory holding costs may have a systematic bias across industries.

The errors in measuring the betas of demand and the systematic bias in estimating inventory holding costs across industries could affect the regression results. It is likely that the betas of demand for firms in the same industry are nearly the same but could differ across industries. Hence, instead of using our estimates of the betas of demand, dummy variables can be used to adjust for the differences in the betas of demand across industries. The systematic bias in estimating inventory holding costs could also result in different intercepts across industries. Dummy variables could be used to adjust for the differences in the intercepts across industries. The unlevered stock betas are regressed on the operating leverage from holding inventories and industry dummy variables,

$$\begin{aligned}\ln \beta =& a_1 + a_2 D_{5311} + a_3 D_{5331} + a_4 D_{5912} \\ &+ c_1 \ln(1 - \frac{h\bar{I}}{(P-C)(\bar{D} - \lambda \mathrm{Cov}(\tilde{D}, \tilde{R}_m))}) + \tilde{e},\end{aligned} \tag{2.30}$$

where the dummy variable for Department Stores (5311), D_{5311}, is defined as equal to 1 for Department Stores, and equal to 0 for others. The dummy variables for Variety Stores (5331) and Drug Stores (5912) are similarly defined. [20] The estimates of the coefficients for this regression equation are:

$a_1 = -1.77$ (standard error is 0.181)

$a_2 = 0.31$ (standard error is 0.219)

$a_3 = 0.42$ (standard error is 0.313)

$a_4 = 0.86$ (standard error is 0.281)

$c_1 = -1.95$ (standard error is 3.528)

The F-value and the adjusted R^2 are 5.252 and 0.2429, respectively.

The estimates of the intercept and the coefficient of the dummy variable for Drug Stores, a_4, are significantly different from zero at the 0.01 level. The estimates of the coefficient of the dummy variables for Department Stores, a_2, and Variety Stores, a_3, are not significantly different from zero at any reasonable level

[20] The coefficient a_1 measures the intercept for Grocery Stores (5411), the coefficient of D_{5311} measures the differences in the intercepts between Department Stores and Grocery Stores, the coefficient of D_{5331} measures the differences in the intercepts between Variety Stores and Grocery Stores, and the coefficient of D_{5912} measures the differences in the intercepts between Drug Stores and Grocery Stores.

of significance. Since dummy variables measure the differences between the intercepts, this suggests that the intercepts across industries are not the same.

The effect of the differences in the intercepts across industries on the coefficient of the operating leverage from holding inventories, c_1, is significant. Although the sign of this coefficient is negative as predicted, the estimate is not significantly different from either zero (t-value is -0.551) or from its hypothesized value of -1 (t-value is -0.27). In the earlier regression without dummy variables the estimate of this coefficient is significantly different from zero, but is not significantly different from its hypothesized value of -1. Given these results, one would conclude that the differences in the intercepts across industries explain most of the unlevered betas and that inventories weakly influence the betas. It seems that inventories could have a "second-order" influence on the beta of the firm. Alternatively, there is not enough variation in the operating leverage from holding inventories in the sample of firms used in this study. Hence, the effect of operating leverage from holding inventories on the beta of the firm is not measured precisely (the standard error of the estimate of c_1 is too big).

2.5. Summary

This chapter analyzes inventory decisions using value maximization as the decision criterion. It focuses on the cash flows and the risk implications of investment in inventories on the value of the firm. A valuation model of the firm is combined with the capital asset pricing model and the option pricing model to derive expressions for the firm's value and the risk from holding inventories. I show that the risk, and hence, the opportunity cost of capital of investments in inventories is an increasing function of the level of investments in inventories. The optimal inventory level for a value maximizing firm is derived. The value maximizing inventory level is a decreasing function of the risk of demand, where the risk of the demand is measured by its covariability with the return on the portfolio that consists of all risky assets in the market. The implications of the results on the procedures for justifying investments that help reduce inventories are discussed. Finally, the notion that holding inventory creates operating leverage similar to the leverage from the commitment of fixed costs is discussed. The higher the level of inventories, the more levered is the firm, and therefore, more risky. The results from an empirical study weakly support the hypothesis that firms holding high inventories are more risky.

CHAPTER 3.

Risk Aversion, Inventories, and Service levels: An Equilibrium Analysis

3.1. Introduction

Firms holding inventories to meet uncertain demand face two types of risks. These are the risks of overstocking and understocking. If demand is less than anticipated firms are left with excess inventory on which holding cost is incurred. If demand is greater than anticipated, there are costs associated with stockouts including the opportunity cost of lost sales, implicit cost of goodwill and future demand, and costs associated with expediting and special processing of backordered demand. Owners of firms balance these risks and choose inventory levels to maximize their expected utility. The utility function depends on the owner's attitude towards risk. Owners with differing attitudes toward risk will stock differently. The level of inventory held by the firm determines the probability of product availability, which is a measure of the level of service provided by the firm to its customers. If the product offered by the firms is the same in terms of quality, attributes, performance characteristics and customers do not perceive any differences in the product across firms, then customers would choose among firms based on the service component of the bundle, that is, the probability of the product availability. The supply of service by firms in the market would be a function of price, cost, demand uncertainty and the risk aversion of the owners of the firms. This chapter examines the effect of risk aversion of the owners of the firms on the equilibrium price and service level in a competitive market.

Gould (1978) develops an equilibrium model of price and service level under the assumption of risk neutrality and shows that firms use inventories as a substitute for a central market. He also shows why multiple prices can exist in the market and why firms use vertical integration and other marketing devices as a substitute for a central market. This chapter extends Gould's model to consider the effect of risk aversion of the owners of the firms on the equilibrium price and service level. It develops a model of market equilibrium where firms face stochastic demand, sell a single product, know their cost function with certainty and the owners maximize the expected utility of profits. In developing the model, I assume that all firms in the market are organized as sole proprietorships and that proprietors are risk averse. The existence of an equilibrium is demonstrated. The following two results are derived. First, if the degree of risk aversion is held constant, an increase in the selling price leads to a higher sevice level or a higher probability of product availability. Second, if the price is held constant, an increase in the degree of risk aversion leads to a decrease in the service level. These results imply that the best price-service combinations in the market would be provided by the least risk averse firms. This suggests that more risk averse firms would not survive in the market.

Nevertheless, firms having different risk characteristics do operate and survive in the market. Two possible explanations for the survival of firms with different risk characteristics are discussed. The first explanation is based on market imperfections such as the extent of information consumers have about firms and the transaction costs incurred by consumers in contacting a firm. The second explanation is based on the risk preferences of the owners and the demand for different price-service combinations by different consumer market segments. I argue that market segments in which consumers are willing to pay higher prices for higher service levels offer investment opportunities for higher returns and higher risks. Correspondingly, the low price-service market segments offer investment opportunities for lower returns and lower risks. I argue that less risk averse firms would dominate the high price-service market segments whereas more risk averse firms would dominate the low price-service market segments.

Section 3.2 describes the market setting. The supply equilibrium is considered in sections 3.3. Section 3.4 describes the demand side of the market. The industry equilibrium is considered in section 3.5. Section 3.6 presents the conclusions of the chapter and suggests directions for future research.

3.2. The Market Setting

The market setting model described below is similar Gould's model. Following Gould (1978, p.3), the underlying assumptions and other important features of the model are:

1. Customers search only one firm and purchase exactly one unit of the product at the price p provided the chosen firm has sufficient inventory. If the firm "stocks out" (i.e., the firm has stocked s units and $i > s$ customers arrive at his store), the firm waits for all the customers to arrive before drawing a lottery. All the arriving customers participate in a lottery, in which each has a positive probability s/i of obtaining the product.

2. Customers know the price and the number of firms in the market, but have no ex ante information about the inventory of any firm or information on how many customers will arrive at a particular firm. If there are M firms, each customer chooses a firm randomly with a probability $1/M$.

3. Each firm in the industry is organized as a sole proprietorship (i.e., the firm is owned and managed by an individual). The owners of the firms are risk averse and have the same utility function. [1] Each firm chooses inventory level at the begining of the period to maximize the expected utility of profits. Inventory decisions are made before the actual demand is known. If the certainty equivalent of the expected utility of profits is positive, firms enter the industry and they leave the industry if the certainty equivalent is negative. [2]

4. Entry and exit do not involve any cost.

Suppose there are M firms and N customers in the market. Given the above assumptions, the probability π_i, that any given firm will have i customers arriving at the firm in the market period is a Binomial distribution and is

$$\pi_i = \frac{N!}{i!\,(N-i)!}\mu^i(1-\mu)^{N-i}$$

where $\mu = 1/M$. If we assume that N and M are large, then π_i can be approximated by a Poisson probability:

$$\pi_i \approx e^{-\lambda}\frac{\lambda^i}{i!}$$

The advantage of this is that π_i can be parametrized in terms of the density of customers per store λ, where $\lambda = N/M$. Entry and exit can therefore be

[1] This assumption also means that the owners of firms have the same degree of risk aversion and display the same attitudes towards risk i.e., decreasing, constant or increasing risk aversion.

[2] See Keeney and Raiffa (1976, ch 4) for a lucid discussion on utility theory and risk. Also see Pratt (1964), Arrow (1965), and Rothschild and Stiglitz (1970, 1971) for more details.

represented by the changes in the parameter λ, which increases as firms leave the industry and decreases as firms enter the industry.

3.3. The Supply Equilibrium

Suppose that the firm faces an exogenous price p and exogenous stochastic demand, which has a Poisson distribution with mean λ. Then the probability that i customers will arrive at the firm in the market period is

$$\pi_i = e^{-\lambda} \frac{\lambda^i}{i!} \tag{3.1}$$

The firm ex ante makes a choice of stocking s units at the cost of $C(s)$. It is assumed that $C'(s) > 0$ and $C''(s) > 0$, that is, the marginal cost is positive and increasing.[3] Assume that all unsold stock perishes. The profit P when s units are stocked at the beginning of the market period is

$$P = \begin{cases} pi - C(s), & \text{if } i \leq s; \\ ps - C(s), & i > s. \end{cases} \tag{3.2}$$

Let $U(P)$ denote the owners's utility when profit is P. Assume that $U'(P) > 0$ and $U''(P) < 0$, that is, the owner of the firm is risk averse. To simplify the notation, we define $U(i, s)$ as the utility when profit is $P = pi - C(s)$. [4] Then the owner's expected utility of profit when s units are stocked and λ is the density of customer per firm is

$$EU(s, \lambda) = \sum_{i=0}^{s} U(i, s) e^{-\lambda} \frac{\lambda^i}{i!} + U(s, s)(1 - F(s)) \tag{3.3}$$

[3] The assumption of increasing marginal cost makes the mathematical analysis relatively uncomplicated. The case of constant marginal cost is discussed later.

[4] Profit is a function of the demand i and the inventory s and therefore the dependence of U on i and s. This simplifies the notation while developing the mathematics. We will also use $U(P)$ in this chapter to denote utility of profit P.

where

$$F(s) = \sum_{i=0}^{s} e^{-\lambda} \frac{\lambda^i}{i!} \tag{3.4}$$

is the probability that demand is less than or equal to s.

Let $MC(s+1) = C(s+1) - C(s)$ be the marginal cost of the $(s+1)st$ unit. Clearly, if $MC(s+1)$ exceeds the price p, the $(s+1)st$ unit would not be stocked even if it could be sold with certainty. Let $\hat{s}$ be the largest s such that $MC(\hat{s}+1) > p$. [5]

Let $MEU(s,\lambda) = EU(s+1,\lambda) - EU(s,\lambda)$, be the the marginal expected utility of profit from one more unit of inventory when the inventory is s. Lemmas 3.1 and 3.2 show that the marginal expected utility is a decreasing function of s.

Lemma 3.1. *If $U(i,s)$ is concave in profits and the marginal cost is increasing then for $i \le s$, the following is true:*

(1). $U(i,s) > U(i,s+1)$

(2). $U(i,s+1) - U(i,s+2) > U(i,s) - U(i,s+1)$

(3). $U(s+1,s+1) - U(s,s) > U(s+2,s+2) - U(s+1,s+1)$

The proof follows immediately from the concavity of $U(i,s)$ and from the fact that marginal cost is increasing.

Lemma 3.2 If $\pi_i > 0$ for $i = 0,1,2\ldots$, then $MEU(s,\lambda) > MEU(s+1,\lambda)$.

Proof: See Appendix B.

It follows from Lemma 3.2 that $EU(s,\lambda)$ is concave in s and the maximum of $EU(s,\lambda)$ occurs at the nonnegative integer s^* for which

$$EU(s^*,\lambda) - EU(s^*-1,\lambda) > 0 \tag{3.5}$$

[5] For the rest of this chapter it is implicitly assumed that $s \le \hat{s}$.

and

$$EU(s^* + 1, \lambda) - EU(s^*, \lambda) \leq 0 \tag{3.6}$$

Note that $s^* \leq \hat{s}$. The uniqueness of s^* is also assured since $MEU(s, \lambda)$ is a decreasing function of s.

For fixed s, $EU(s, \lambda)$ is a concave function of λ (see Appendix B). Let $\hat{P}$ be the *certainty equivalent* associated with $EU(s, \lambda)$, that is, $U(\hat{P}) = EU(s, \lambda)$. For fixed s, $\hat{P}$ is also a concave function of λ. When λ tends to zero $\hat{P}$ tends to $-C(s)$ and when λ tends to infinity $\hat{P}$ tend to $ps - C(s)$. [6] For fixed s, $\hat{P}$ looks like Figure 3.1, when drawn as a function of λ. Whenever $ps - C(s) > 0$ there exists a value of λ such that $\hat{P} = 0$. Since at equilibrium the *certainty equivalent* $\hat{P}$ for each firm in the industry should be equal to zero, we are interested in the conditions associated with this λ.

The supply equilibrium conditions are such that firms choose s^* to maximize expected utility of profits and λ is such that the *certainty equivalent* of the expected utility is zero. Thus, we want to find s^* and λ such that

$$U(0) = EU(s^*, \lambda) \tag{3.7}$$

and conditions (3.5) and (3.6) are satisfied simultaneously. Conditions (3.5) and (3.6) are the optimality conditions for maximizing the expected utility for a given λ. If (3.7) is not satisfied the market is in disequilibrium and entry and exit of firms will change λ. The market would be in equilibrium only if (3.5), (3.6), and (3.7) are satisfied simultaneously. The following lemmas will be used to characterize the supply equilibrium.

[6] This can be easily seen from equation (3.3). As λ tends to zero, the probability that demand is zero tends to 1. Hence, the expected utility equals $U(-C(s))$. On the other hand, as λ tends to infinity, the probability that demand is greater than s tends to 1. Hence the expected utility equals $U(ps - C(s))$.

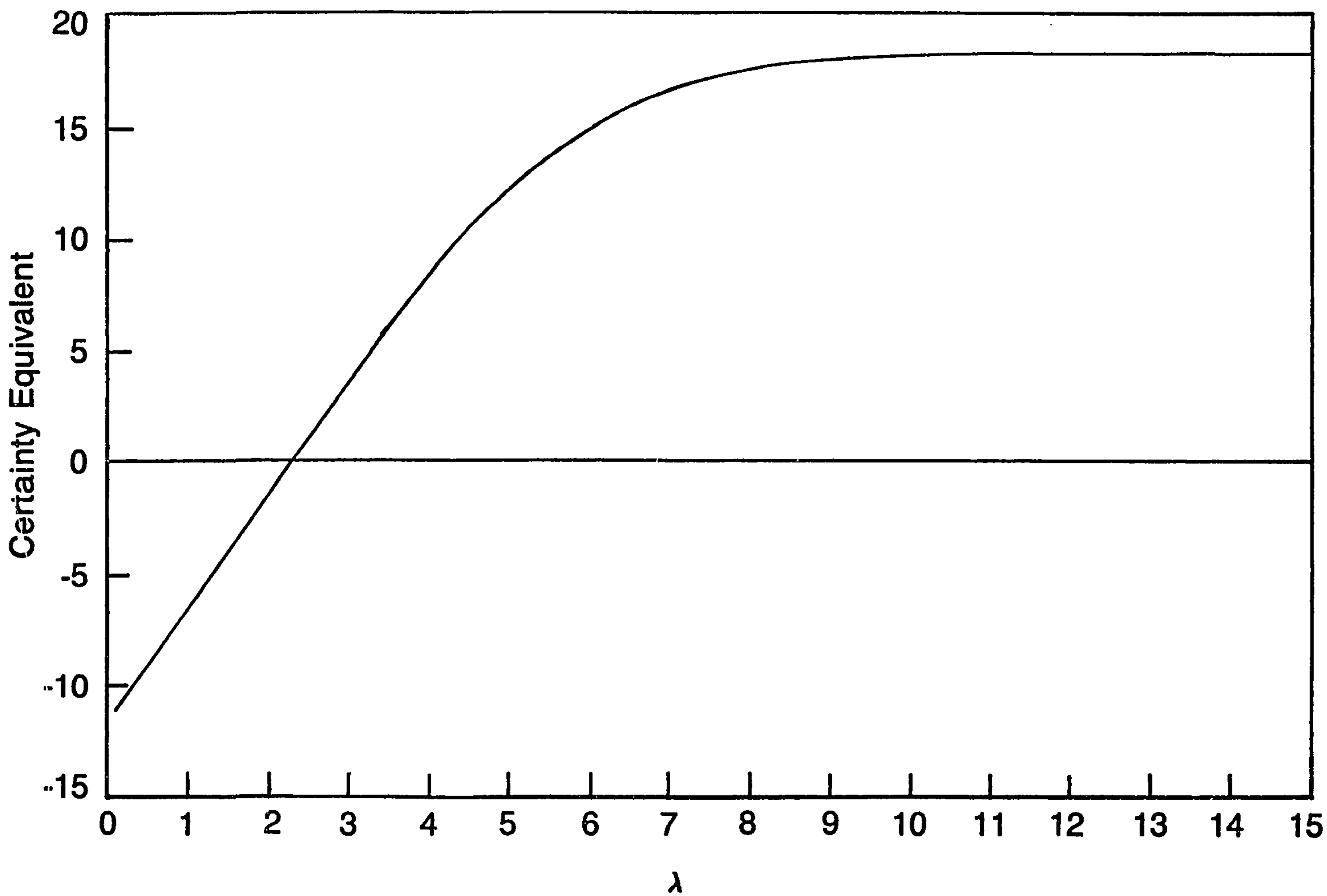

Figure 3.1. Certainty equivalent of expected utility, as a function of λ, for fixed stock level s.

Lemma 3.3. *for every s such that $p > MC(s+1)$ there exist a unique λ (denoted by λ_s) such that $EU(s,\lambda) = EU(s+1,\lambda)$.*

Proof:

$$EU(s+1,\lambda) - EU(s,\lambda) = \sum_{i=0}^{s}(U(i,s+1) - U(i,s))e^{-\lambda}\frac{\lambda^i}{i!}$$
$$+ (U(s+1,s+1) - U(s,s))(1 - F(s))$$

Therefore, $EU(s,\lambda) = EU(s+1,\lambda)$ when

$$F(s,\lambda) = 1 - \frac{\sum_{i=0}^{s}(U(i,s) - U(i,s+1))e^{-\lambda}\frac{\lambda^i}{i!}}{U(s+1,s+1) - U(s,s)} \tag{3.8}$$

The following can be easily shown: [7]

(1). $F(s,0) = 1$

(2). $F(s,\lambda)$ is a monotonically decreasing function of λ.

(3). $\lim_{\lambda\to\infty} F(s,\lambda) = 0$.

(4). When $\lambda = 0$, $1 - \frac{\sum_{i=0}^{s}(U(i,s)-U(i,s+1))e^{-\lambda}\frac{\lambda^i}{i!}}{U(s+1,s+1)-U(s,s)} < 1$.

(5). $1 - \frac{\sum_{i=0}^{s}(U(i,s)-U(i,s+1))e^{-\lambda}\frac{\lambda^i}{i!}}{U(s+1,s+1)-U(s,s)}$ is a monotonically increasing function of λ

(6). $\lim_{\lambda\to\infty} 1 - \frac{\sum_{i=0}^{s}(U(i,s)-U(i,s+1))e^{-\lambda}\frac{\lambda^i}{i!}}{U(s+1,s+1)-U(s,s)} = 1$.

Since $F(s,\lambda)$ decreases in λ and the right hand side of (3.8) increases in λ they must intersect as λ increases. Therefore, the solution exists and is unique.

Lemma 3.4. *If $s_1 > s_2$ and if both λ_{s_1} and λ_{s_2} are defined then $\lambda_{s_1} > \lambda_{s_2}$.*

Proof: From equation (3.8) we have

$$F(s,\lambda) = 1 - \frac{\sum_{i=0}^{s}(U(i,s) - U(i,s+1))e^{-\lambda}\frac{\lambda^i}{i!}}{U(s+1,s+1) - U(s,s)}.$$

[7] Note the modification in the notation making the dependence of $F(.,.)$ explicit on λ as well as s.

From Lemma 3.1 we can see that $U(s+1,s+1) - U(s,s)$ decreases in s and $U(i,s) - U(i,s+1)$ increases in s. Hence, the right hand side of (3.8) decreases as s increases. Since $F(s,\lambda)$ increases in s and decreases in λ and the right hand side of (3.8) is an increasing function of λ, λ must increase to maintain the equality. Hence $\lambda_{s_1} > \lambda_{s_2}$.

Lemma 3.5. *$MEU(s,\lambda)$ is a continuous monotonically increasing function of λ.*

Proof:

$$\begin{aligned} MEU(s,\lambda) = &\sum_{i=0}^{s}(U(i,s+1) - U(i,s))e^{-\lambda}\frac{\lambda^i}{i!} \\ &+ (U(s+1,s+1) - U(s,s))(1 - F(s)) \end{aligned}$$

$$\begin{aligned} \frac{\partial MEU(s,\lambda)}{\partial\lambda} = &- \sum_{i=0}^{s}(U(i,s+1) - U(i,s))e^{-\lambda}\frac{\lambda^i}{i!} \\ &+ \sum_{i=1}^{s}(U(i,s+1) - U(i,s))e^{-\lambda}\frac{\lambda^{i-1}}{i-1!} \\ &+ (U(s+1,s+1) - U(s,s))e^{-\lambda}\frac{\lambda^s}{s!} \\ = &- \sum_{i=0}^{s}(U(i,s+1) - U(i,s))e^{-\lambda}\frac{\lambda^i}{i!} \\ &+ \sum_{i=0}^{s-1}(U(i+1,s+1) - U(i+1,s))e^{-\lambda}\frac{\lambda^i}{i!} \\ &+ (U(s+1,s+1) - U(s,s+1))e^{-\lambda}\frac{\lambda^s}{s!} \end{aligned} \quad (3.9)$$

From Lemma 3.1 we have $U(i+1,s+1) - U(i+1,s) > U(i,s+1) - U(i,s)$ and $U(s+1,s+1) > U(s,s+1)$. Hence, $MEU(s,\lambda)$ is a continuous monotonically increasing function of λ.

Lemma 3.6.

$$\lambda > \lambda_s \Rightarrow EU(s+1,\lambda) > EU(s,\lambda)$$

$$\lambda < \lambda_s \Rightarrow EU(s+1,\lambda) < EU(s,\lambda)$$

Proof: Using Lemma 3.5 and the fact that

$$MEU(s,0) = U(-C(s+1)) - U(-C(s))$$

$$MEU(s,\lambda_s) = 0$$

and

$$\lim_{\lambda\to\infty} MEU(s,\lambda) = U(s+1,s+1) - U(s,s)$$

this Lemma can be easily proved.

Theorem 3.1.

$$s^* = s \Leftrightarrow \lambda_{s-1} < \lambda \leq \lambda_s.$$

Proof: $\Rightarrow$. If $s^* = s$ then by Lemma 3.2 $EU(s,\lambda) > EU(s-1,\lambda)$ and $EU(s+1,\lambda) \leq EU(s,\lambda)$. From Lemma 3.6 this implies that $\lambda > \lambda_{s-1}$ and $\lambda \leq \lambda_s$.

$\Leftarrow$. If $\lambda > \lambda_{s-1}$ then by Lemma 3.6 $EU(s,\lambda) > EU(s-1,\lambda)$, and similarly if $\lambda \leq \lambda_s$ then $EU(s+1,\lambda) \leq EU(s,\lambda)$ (with equality holding only when $\lambda = \lambda_s$). Thus by Lemma 3.2 we have $s^* = s$.

Theorem 3.1 enables us to construct s^* as a function of λ. We can first find the set $\{\lambda_i\}$ $i = 0,1,\ldots,\hat{s}+1$ where λ_i satisfies $EU(s,\lambda) = EU(s+1,\lambda)$. Then for any λ we can locate the half-open interval $(\lambda_{i-1},\lambda_i]$ which contains λ. By Theorem 3.1 $s^* = i$ for all $\lambda \in (\lambda_{i-1},\lambda_i]$. $EU(s^*(\lambda),\lambda)$ can now be constructed and is the envelope of the curves $EU(s,\lambda)$ *for* $i = 0,1,\ldots,\hat{s}+1$. Note that $EU(s,\lambda)$

dominates all other expected utility curves when $\lambda \in (\lambda_{s-1}, \lambda_s]$. $EU(s^*(\lambda), \lambda)$ can be viewed as the dominating curve over all λ. Figure 3.2 graphs $EU(s^*(\lambda), \lambda)$. In order to find the equilibrium values of s^* and λ we need to find the value of λ where $EU(s^*(\lambda), \lambda) = U(0)$. The equilibrium value in Figure 3.2 exists and is indicated as λ_e. [8]

The lemmas and theorem proved in this section enable us to characterize the supply equilibrium. Suppose the price is fixed and the stock level is i. Then there exist an interval $(\lambda_{i-1}, \lambda_i,]$ such that for any $\lambda \in (\lambda_{i-1}, \lambda_i]$ expected utility at inventory i is greater than expected utility at any other inventory, that is, the expected utility at inventory i dominates the expected utility at any other inventory level. If the equilibrium value of $\lambda \in (\lambda_{i-1}, \lambda]$, then the equilibrium stock level will be i. For each stock level we can find the dominating interval. We can then construct the dominating expected utility curve as a function of stock level and λ. The point where the dominating expected utility curve is equal to $U(0)$ gives the equilibrium values of stock and λ.

Supply Equilibrium Characteristics

The interesting feature of the supply equilibrium is that the equilibrium values of the density of customer per firm, λ, and the stock level, s, depends not only on the price p and the cost function $C(.)$ but also on the utility function of the owners of the firms. Given the equilibrium values of λ and s, the equilibrium probability π_e that any customer gets the product is

$$\pi_e = F(s-1) + \frac{s}{\lambda}(1 - F(s)). \tag{3.10}$$

where

$$\pi_e(s+1, \lambda) - \pi(s, \lambda) = \frac{1 - F(s+1, \lambda)}{\lambda} > 0. \tag{3.11}$$

[8] The utility function used to draw Figure 3.2 is $U(P) = -e^{-rP}$.

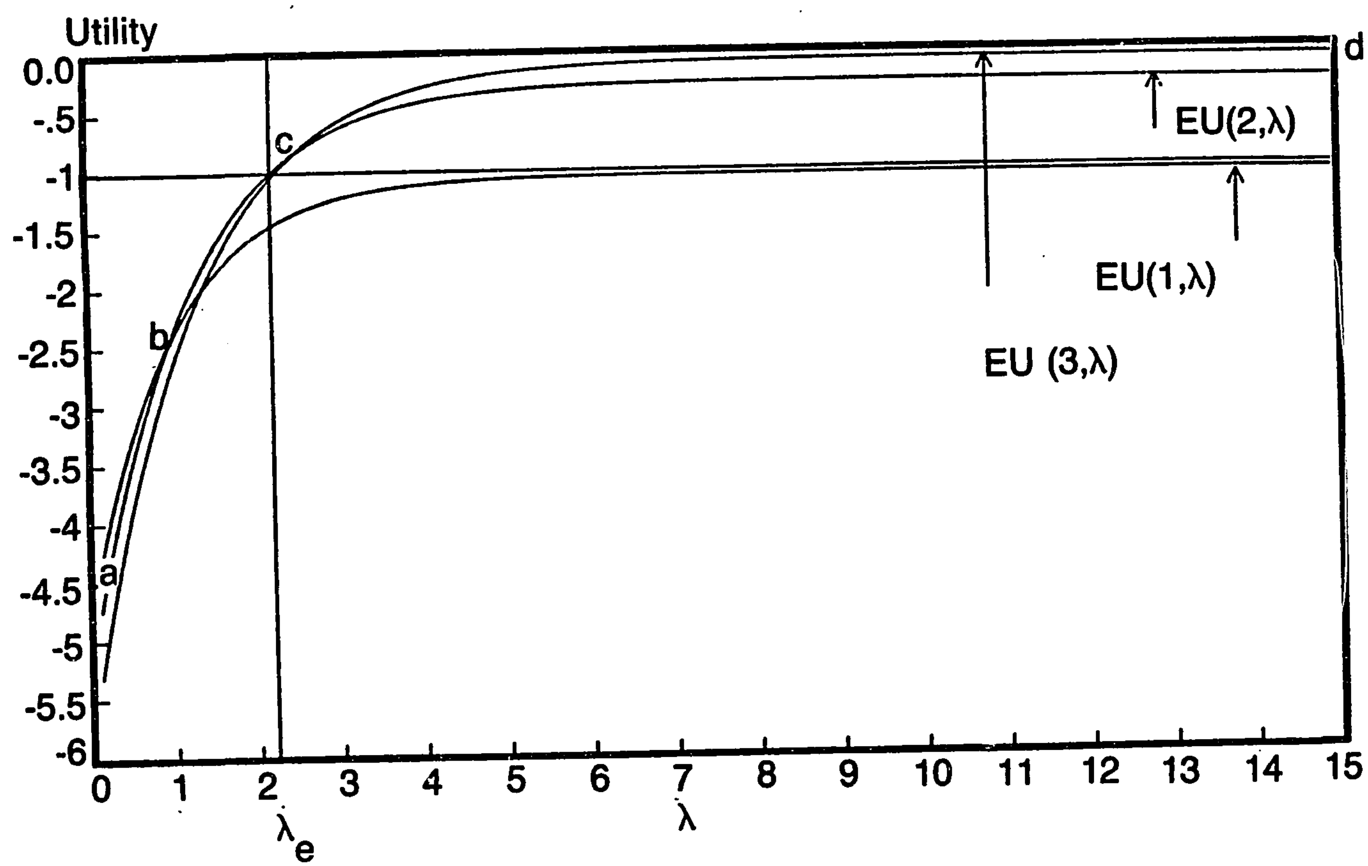

Figure 3.2. Expected utility curves as a function of λ, for various values of stock s. The utility function used is $U(P) = -e^{-rP}$. The dominating curve is given by "abcd", and the equilibrium stock and density of customer per firm values is given by the point where the dominating curve equals -1.

and [9]

$$\frac{\partial \pi_e}{\partial \lambda} = -\frac{s}{\lambda^2}(1 - F(s,\lambda)) < 0. \tag{3.12}$$

π_e can be viewed as the service level provided by the industry under equilibrium. Among other things π_e depends on the utility function and therefore, on the owner's attitude towards risk. Since, we are concerned with the effect of risk aversion on the supply of service in the market, it will be useful to discuss the notion of measuring risk aversion to indicate when one owner is more risk averse than another.

A measure of risk aversion is the local risk aversion function $r(P)$ defined as

$$r(P) = -\frac{U''(P)}{U'(P)}$$

Let U_1 and U_2 be utility functions with local risk aversion r_1 and r_2, respectively. If, at a point P, $r_1(P) > r_2(P)$ then U_1 is locally more risk-averse than U_2 at point P. If, $r_1(P) > r_2(P)$ for all P, that is, U_1 has greater local risk aversion than U_2 everywhere, then U_1 is also globally more risk-averse than U_2, in the sense that, for every risky outcome the certainty equivalent is always less with utility function U_1 than with U_2. Equivalently, the risk premium (expected monetary value minus the certainty equivalent) is always larger with utility function U_1 than with U_2.

In the discussion that follows, the use of the term "more risk-averse" or "less risk-averse" should be interpreted as more or less risk averse in the global sense. Let r denote the degree of risk aversion of the owner of the firm. If owners of Firm A and B have degree of risk aversion r_1 and r_2 respectively, and if $r_1 > r_2$, then, the owner of Firm A is globally more risk-averse than the owner of Firm B. Define the vector (r, p, π) as the equilibrium price-service combination when all

9 See Gould (1978) for the proof of (3.10), (3.11), and (3.12).

firms have the same utility function and same degree of risk aversion r. I next examine the relationship between π to changes in p and r.

The Relationship Between π_e and p

Let s_e and λ_e be the equilibrium values of stock and density of customers per firm respectively, at an exogenous price p and degree of risk aversion r. *Ceteris paribus*, an increase in price increases the expected utility of profits of firms, which in turn will increase the certainty equivalent of profits $\hat{P}$ from zero to some positive level. Since, the certainty equivalent of profits is positive the market is in disequilibrium. If firms do not change the stock level, then λ_e must decrease to reduce the certainty equivalent of profits to zero (recall that $\frac{\partial EU(s,\lambda)}{\partial \lambda} > 0$.) The decrease in λ_e will be such that $\hat{P}$ will be zero and the market will be in equilibrium. Therefore, λ_e is a decreasing function of price. This is intuitive since a higher price would attract more firms in the industry, which will decrease λ_e and increase the probability of product availability π_e.

Under the assumptions of risk-neutrality Gould (1978) has shown that as price increases, s_e remains the same or decreases. I give an intuitive argument for the above and use it to show that risk averse firms will display similar behavior. Recall that an increase in price leads to a decrease in λ_e. If λ_e decreases, the probability that demand is less than s_e increases. This means that the probability of firms being left with unsold stocks at the end of the period increases, which increases the probability of incurring a loss. On the other hand, an increase in price increases the profit level. If risk neutral firms find it optimal to decrease stock with an increase in price it must be because the expected loss increases at a faster rate than the rate of increase in expected profits. If firms are risk averse, the decrease in expected utility due to higher expected losses will be more than the increase in utility due to higher expected profits. Therefore, as p increases, risk averse firms will either keep the equilibrium stock levels the same or decrease it. Risk averse firms, in comparison to risk neutral firms, will adjust their stock downward at a higher value of λ.

As p increases, λ decreases and π_e increases. When p reaches a certain critical level, firms adjust their stock downward and π_e decreases since the small decrease in λ, which accompanies the stock adjustment, is not sufficient to keep π_e from rising. Therefore, π_e drops at this point and rises thereafter, until the next downward adjustment. The relation between π_e and p is a "saw-toothed" relationship with π_e taking all values in $(0, 1]$ and $\pi_e \to 1$ as $p \to \infty$. [10]

The Relationship Between π_e and r.

For fixed price p and degree of risk aversion r, let the equilibrium values of stock, density of customers per firm and probability of product availability be s_e, λ_e, and π_e respectively, that is [11]

$$EU(r, s_e, \lambda_e) = U(0).$$

Suppose the degree of risk aversion increases from r to r_1. Then, it is easy to see that $\hat{P}$, the certainty equivalent of $EU(r_1, s_e, \lambda_e)$ is < 0. [12] Since $\hat{P}$ is an increasing function of λ, λ_e will increase (at s_e held constant) so that $\hat{P} = 0$. Therefore, at equilibrium more risk averse firms will have higher values of λ_e

[10] The saw-toothed relation between π_e and p is because the demand is discrete. In the case of continuous demand, π_e will increase as p increases and firms will adjust stock continuously.

[11] Note the modification in the notation making the dependence of $EU(.,.,.)$ and $U(.,.,.)$ explicit on r,s, and λ.

[12] For fixed p, λ_e, and s_e both the firms face the same payoff. If the certainty equivalent of the less risk averse firm is zero then the certainty equivalent of the more risk averse firm will be < 0.

implying that more risk averse firms will provide a lower equilibrium probability of product availability as compared to less risk averse firms. Another implication is that more risk averse firms will offer a given service level at higher price.

If the price is held constant and the risk aversion of firms increases, then s_e remains the same or decreases. Increasing risk aversion will increase λ_e. Holding stock constant at s_e, an increase in λ_e decreases the expected loss and increases the expected profits. When the risk aversion increases, the utility associated with the expected loss decreases at a faster rate in comparison to the the rate of increase in utility due to an increase in expected profits. Therefore, firms will lower s_e as risk aversion increases. π_e decreases as risk aversion increases and will drop further whenever there is a downward adjustment of s_e . Furthermore, $\pi_e \to 0$, as $r \to \infty$. Figure 3.3 shows the nature of the equilibrium probability curves as a function of price and various degrees of risk aversion.

The model has been developed assuming that the marginal cost is increasing. The assumption of constant marginal cost presents no serious difficulties for the model developed in the earlier sections. However, it does present difficulties in constructing the equilibrium values of p and π_e. The reason is that as p gets smaller s^* is not bounded from above as it is when the marginal cost is increasing. This means that as p decreases both s^* and λ rise. The result is that $\pi_e \to 1$ as $p \to c$, where c is the constant marginal cost. Therefore, the best price-service combination is offered by a very large firm and there is a tendency for a single large firm with low risk aversion to dominate.

Computational Results

The numerical values of π_e, λ_e, and s_e for different prices and various values

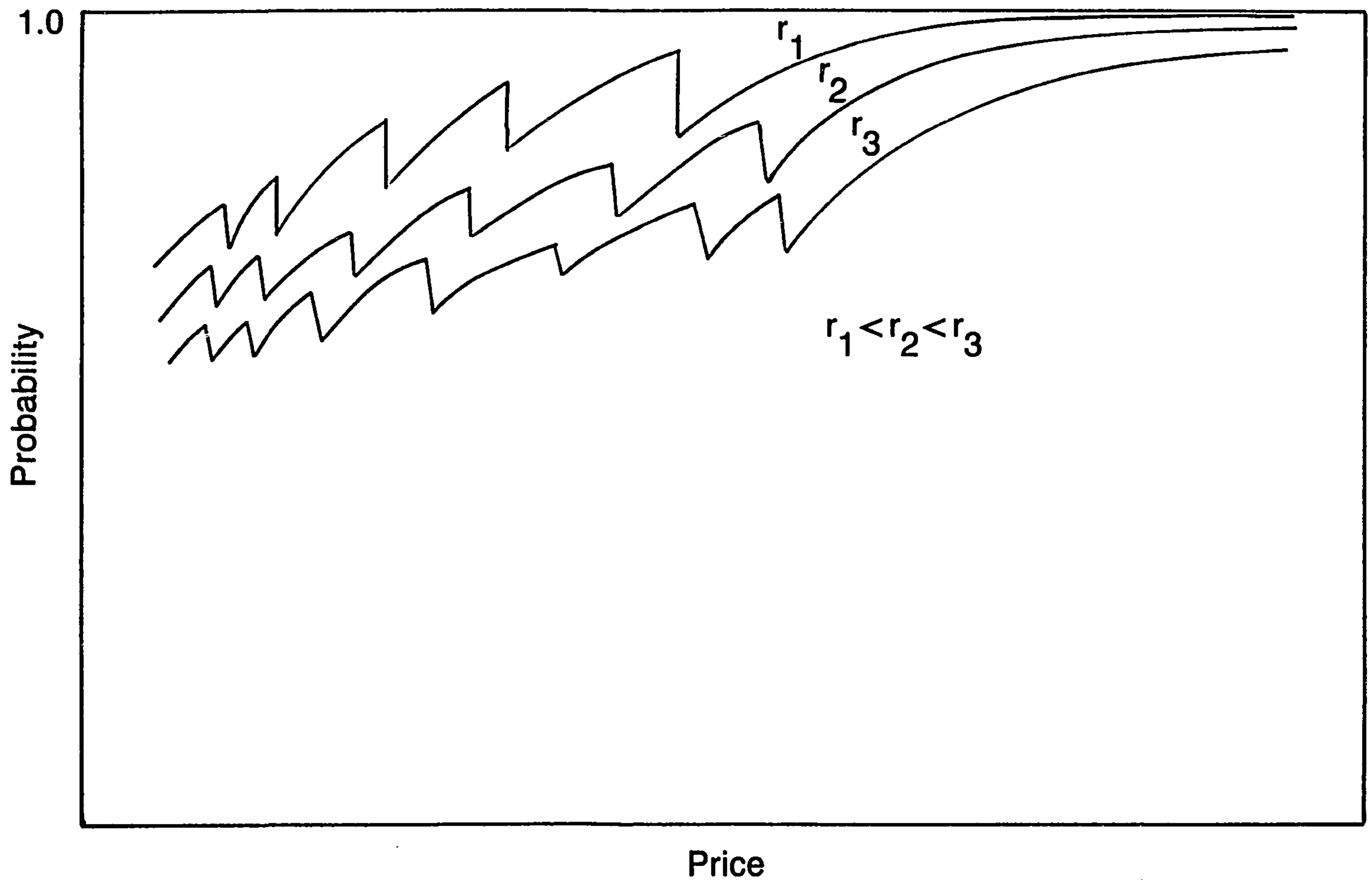

Figure 3.3. Equilibrium probability as a function of price for various values of the degree of risk aversion r.

of the degree of risk aversion r, are presented in Table 3.1. The equilibrium values are computed using the following.

(1). The utility function is $U(P) = -e^{-rP}$, with r being the degree of risk aversion.

(2). The cost function is $C(s) = \frac{s^2}{10} + \frac{s}{4} + 10$.

(3). For each p and r, the utility function is computed for $s = 1, 2, \ldots, 15$ and λ varying from .1 to 15 in increments of .1. For each λ the value of s and the value of expected utility which dominate all other expected utility curves is found so that the dominating expected utility curve as a function of s^* and λ can be generated. The point where the dominating curve equals -1 ($U(0) = -1$), gives the equlibrium values of s_e and λ_e because this is the condition for competitive entry.

(4). The degree of risk aversion is varied from 0 to 1 in increments of .10.

From Table 3.1 we observe the following:

(1). At constant price, as risk aversion increases, λ_e increases, s_e remains the same or decreases and π_e decreases.

(2). At constant risk aversion, as price increases, λ_e decreases, s_e remains the same or decreases. In the range of prices where s_e remains the same we find that π_e increases. When s_e drops we see that π_e drops and then starts to increase once again with further increases in price.

(3). The best equilibrium price service combination at any price is provided by the least risk averse firms. (In this example by the risk neutral firm, that is, $r = 0$)

Table 3.1

Equilibrium Values of Stock (s), Density of Customers per Firm(λ), and Probability of Product Availability (π_e), for Various Prices (p) and Degree of Risk Aversion (r).

DEGREE OF RISK AVERSION	PRICE						
	3.30	4.30	6.30	7.00	8.00	9.00	10.00
0.00	(5, 5.1, .82)	(4, 3.4, .86)	(4, 2.1, .96)	(3, 1.9, .91)	(3, 1.6, .94)	(3, 1.4, .95)	(3, 1.3, .96)
0.10	(5, 5.7, .77)	(4, 4.0, .80)	(3, 2.6, .82)	(3, 2.4, .85)	(3, 2.1, .87)	(3, 1.9, .90)	(3, 1.9, .91)
0.20	(5, 6.3, .72)	(4, 4.6, .75)	(3, 3.3, .74)	(3, 3.3, .74)	(3, 2.9, .78)	(2, 2.8, .62)	(2, 2.6, .64)
0.30	(5, 7.0, .67)	(4, 5.4, .68)	(3, 4.1, .65)	(3, 4.0, .66)	(2, 3.8, .50)	(2, 3.6, .52)	(2, 3.4, .53)
0.40	(5, 7.9, .61)	(4, 6.3, .60)	(3, 5.1, .56)	(2, 4.9, .40)	(2, 4.6, .42)	(2, 4.6, .42)	(2, 4.4, .43)
0.50	(5, 8.8, .56)	(4, 7.2, .54)	(3, 6.1, .48)	(2, 5.8, .34)	(2, 5.6, .35)	(2, 5.6, .35)	(2, 5.5, .36)
0.60	(5, 9.8, .51)	(3, 8.2, .36)	(2, 7.1, .28)	(2, 6.8, .29)	(2, 6.6, .30)	(2, 6.6, .30)	(2, 6.5, .31)
0.70	(5, 10.8, .46)	(3, 9.1, .33)	(2, 8.0, .25)	(2, 7.8, .26)	(2, 7.7, .26)	(2, 7.6, .26)	(2, 7.6, .26)
0.80	(4, 11.9, .34)	(3, 10.1, .30)	(2, 9.0, .22)	(2, 8.8, .23)	(2, 8.8, .23)	(2, 8.7, .23)	(2, 8.7, .23)
0.90	(4, 12.8, .31)	(3, 11.1, .27)	(2, 10.0, .20)	(2, 9.9, .20)	(2, 9.9, .20)	(2, 9.8, .20)	(2, 9.8, .21)
1.00	(4, 13.9, .29)	(3, 12.1, .25)	(2, 11.1, .18)	(2, 11:0, .18)	(2, 10.9, .18)	(2, 10.9, .18)	(2, 10.9, .18)

The numbers within the parenthesis are the equilibrium values of s, λ, and π_e respectively.

Risk aversion = 0 numbers are for the risk neutral firms.

3.4. The Demand Side

The last few sections have developed relations among π, p, r, and λ and defined the vector (r, p, π) an equilibrium if it results in the certainty equivalent of zero for all firms in the industry. To understand which of the infinite set of these supply equilibrium vectors will result in the market we need to say more about the consumer behavior. I use a simple model to describe the behavior of consumers. Consider a consumer, who desires one unit of the product and also sees the possibility that the firm he contacts will have sold out before the product is purchased . If Y is the consumer's income, p the price, h the utility associated with the product and π the probability of product availability then the expected utility of the consumer is

$$E(U) = \pi((U(Y-p)+h)+(1-\pi)(U(Y)). \tag{3.13}$$

Using (3.13) we can establish a reservation price $\hat{p}$, such that for any $p > \hat{p}$ the consumer would not buy the product even if it is available with certainty. Assuming that $U'(.) > 0$ and $U''(.) < 0$, it can be shown that for constant expected utility $\frac{\partial \pi}{\partial p} > 0$ and $\frac{\partial^2 \pi}{\partial p^2} > 0$, and the relevant consumer indifference curves drawn on the (π, p) plane, are convex and sloping upward, with higher indifference curve further up to the northwest. This means that consumers trade off price with service level and are willing to pay higher price for higher service levels.

3.5. The Industry Equilibrium

Figure 3.4 graphically depicts the industry equilibrium. The price-service combinations provided by the least risk averse firm dominates the price-service combinations provided by more risk averse firms. The equilibrium in the

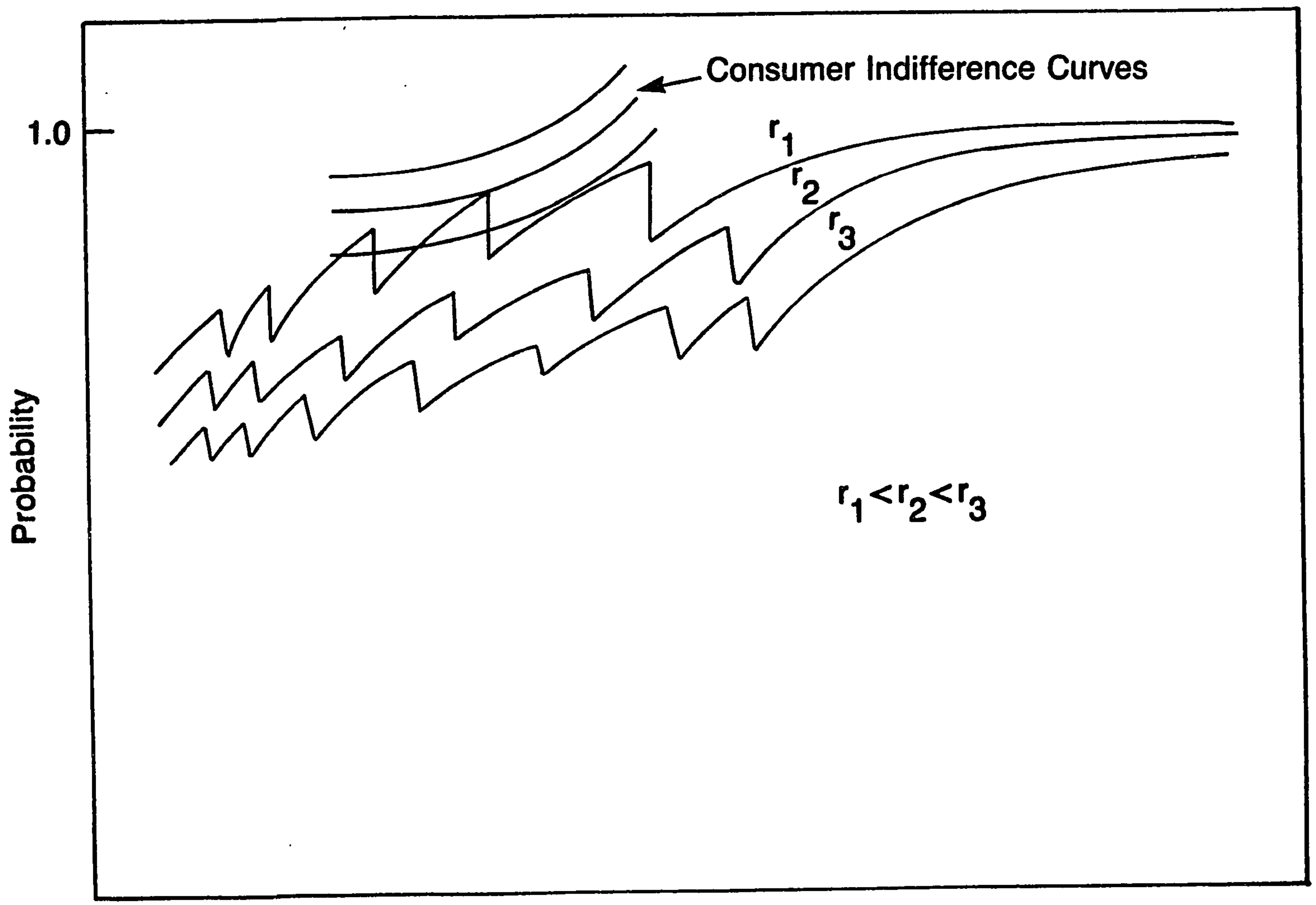

Figure 3.4. Consumer indifference curves, firm's supply curves and industry equilibrium.

industry occurs at the point where the consumer indifference curve is tangent to the supply curve of the least risk averse firms.

The equilibrium analysis of Figure 3.4 indicates that the best price-service combination in the industry would be provided by the least risk averse firms. This suggests that more risk averse firms would not survive in the market. But empirically firms having different risk characteristics do operate and survive in the same industry. There are two possible explanations for the survival of firms with different risk characteristics. The first is based on market imperfections, such as the extent of information consumers have about the firm and the transaction costs incurred by the consumers in contacting a firm. The second explanation is based on the risk preferences of the owners and the demand for different price-service combinations by different consumer segments.

Transaction Costs and Consumer Knowledge

Suppose consumers know the number of firms operating in the industry but have no information on the stock level of the firm. Consumer rationality implies that if there are M firms, consumers choose randomly among firms and the probability of a firm being chosen by a consumer is $1/M$. The random selection of firms by consumers determines λ, the density of customers per firm. Suppose $M-1$ firms out of M have degree of risk aversion equal to r, the Mth firm has a degree of risk aversion $r_1 > r$ and in equilibrium they stock s_r and s_{r1} respectively. If in equilibrium $EU(r, s_r, \lambda) = U(0)$ and $s_r = s_{r1}$, it is easy to see that $EU(r_1, s_{r1}, \lambda) < U(0)$, that is, the certainty equivalent of the expected utility of the Mth firm < 0. [13] Hence, the Mth firm will either leave the industry or can stay by lowering the stock from s_r, (recall that marginal expected utility is a

[13] See footnote 12.

decreasing function of s) so that $EU(r_1, s_{r1}, \lambda) = U(0)$ and $s_{r1} < s_r$. This means that for a given p and λ (fixed by the random selection among M firms), $\pi_r > \pi_{r1}$. The Mth firm could still survive in the industry by offering lower service level at the same price if

(1) consumers do not know the price-service combinations offered by the other $M-1$ firms in the industry, or

(2) if all the other $M-1$ firms raise their price without changing stock and λ, that is, they offer a higher service level at a higher price compared to the Mth firm and consumers are indifferent between the two price-service combinations in the market, or

(3) if the transaction costs incurred by consumers in visiting the Mth firm are different from the other $M-1$ firms and the transaction costs are such that $t_{r1} < t_r$ and

$$t_{r1}(1 - \pi_{r1}) = t_r(1 - \pi_r) \tag{3.14}$$

where t_{r1} is the transaction cost of visiting the Mth firm and t_r is the cost of visiting any of the other $M-1$ firms.

If the transaction costs are the same across all firms, consumers have information about the stock level of the firms and the other $M-1$ firms cannot raise their prices, then the industry will be in equilibrium only if consumers choose the Mth firm with probability $< \frac{1}{M}$. If consumers know the number of firms in the industry and the stock level of each firm, then rationality implies that they choose the store such that the probability of product availability is the same across all firms. [14] This implies choosing θ_r and θ_{r1} (the probability of choosing the two

[14] Suppose there are N consumers and that $N-1$ of them select firms in such a manner that one of the firm has a higher probability of product availability than the other firms. The best strategy for the Nth consumer would then be

different kind of firms) such that $(M-1)\theta_r + \theta_{r1} = 1$ and $\theta_r > \theta_{r1}$. This means that $\lambda_r > \lambda_{r1}$. The *Mth* firm sees a lower density of customer and will respond by lowering the stock, so that the combined effect of lower λ and lower s results in $\pi_r = \pi_{r1}$.

Risk Preferences and Consumer Segments

Suppose in the market there are some consumers who are willing to pay a higher price for a higher probability of product availability, together with some who are willing to accept a lower probability of product availability at a lower price. The high price-service market segment offers investment opportunities for higher expected returns and higher risks. Higher prices increase the expected profit levels and therefore, the expected returns. Higher service level requires a larger inventory, thereby increasing the risk of the firm being left with unsold stock. Correspondingly, the low price-service market segment offers investment opportunities for lower expected returns and lower risks. The risk-return combinations for different price-service combinations will be a concave curve sloping upward, with higher expected return and higher risk associated with higher price-service combinations.

Consider a risk averse owner faced with the expected return and risk opportunities associated with different price-service combinations. The owner's indifference curve between expected return and risk will be convex and positively sloped. A higher degree of risk aversion would mean that the curves are more positively sloped. The optimal risk-return combination for the firm would be at the point where the indifference curves are tangent to the risk-return opportunities provided

to choose this highest probability firm with certainty. This is not rational at the market level because every customer would go to this firm and this firm could no longer have the highest probability.

by the price-service level combinations. Figure 3.5 shows the opportunity set and the indifference curve for firms having different degree of risk aversion. It can be seen from the figure that less risk averse firms find it optimal to select the high price-service level market segment and the more risk averse firms find it optimal to choose the low price-service market segment. Consumers who desire a higher probability of product availability choose the less risk averse firm. Consumers who are willing to pay less for lower service level will choose the more risk averse firm. The less risk averse firm will not find it optimal to enter the low price-service market segment since the risk-return combination from this market segment does not match the firm's risk-return preferences and the firm will end up taking "too little risk".

3.6. Summary

This chapter develops a model of the effect of risk aversion of the owners of the firms on the equilibrium price and service levels in a competitive market. It shows that more risk averse owners stock less and provide lower service levels. More risk averse owners should dominate the low price-service market segments whereas less risk averse owners would find it optimal to cater to the high price-service market segments.

An interesting research issue is to extend the analysis to consider the effect of alternative organizational forms, such as open corporations, on the equilbrium price-service levels. This is of interest because proprietorships are not the only forms of organization that are observed in the market. Furthermore, there is competition among organizational forms for survival. The form of organization that survives is the one that provides the highest service level at the lowest price. Extending the model to consider alternative organizational forms is beyond the

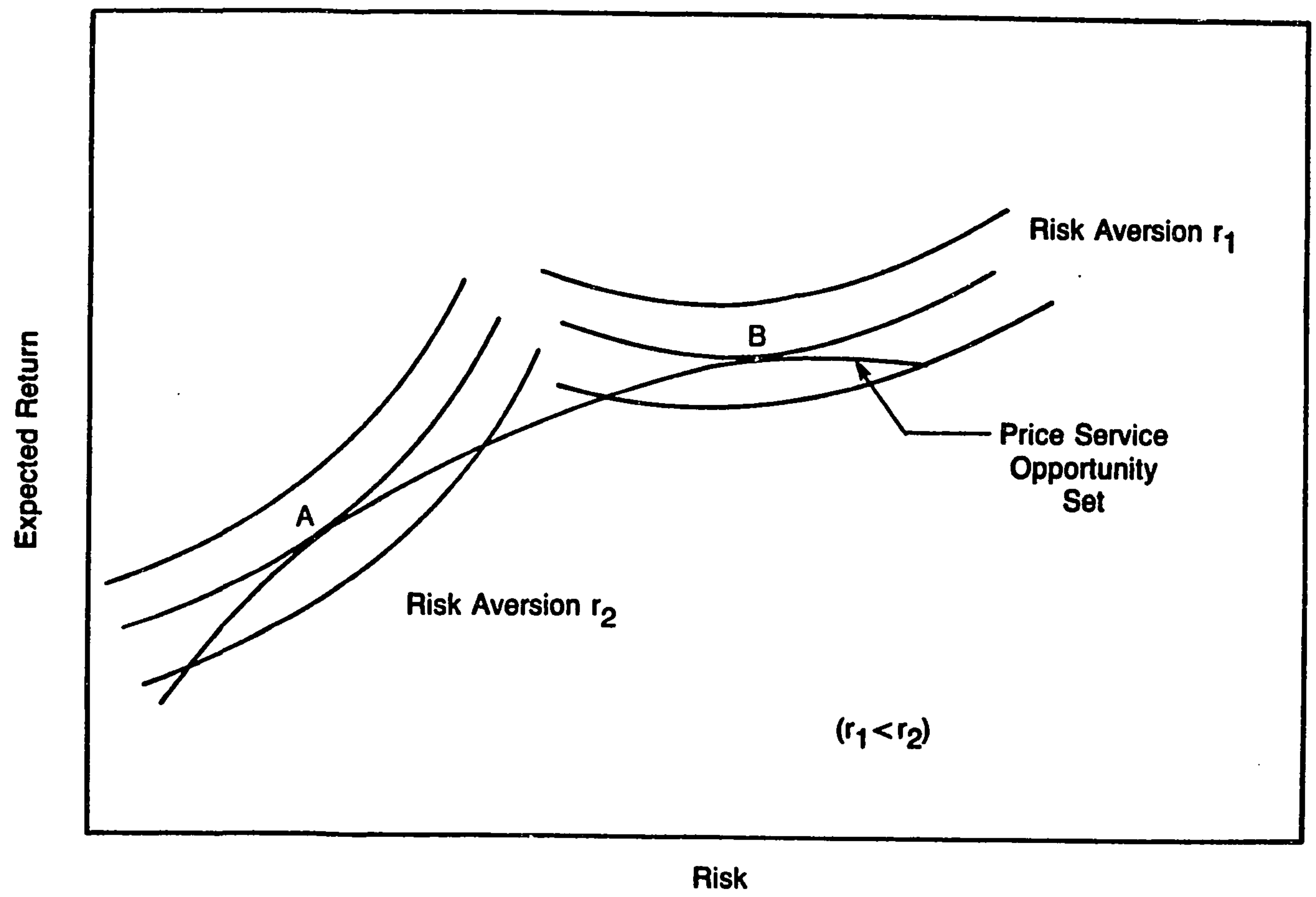

Figure 3.5. The optimal choice of price-service combinations for firms with different degree of risk aversion.

scope of this chapter. However, I briefly discuss some of the important factors that must be considered in extending the model. Specifically, I compare two alternative forms of organization, proprietorships and open corporations.

Proprietorships and open corporations differ from each other in three important ways. First, proprietorships are owned and managed by a single individual. The owner of the firm makes the investment decisions of the firm and bears the risk of his decisions. On the other hand, open corporations are characterized by unrestricted risk sharing among the owners of the firm and by complete separation of risk-bearing and decision-making functions. Fama and Jensen (1983a, 1983b) argue that there are advantages due to specialization in risk-bearing and decision management. The common stock of open corporations allows the risk to be spread across many owners who on their own can choose the level of risk they wish to bear and who can diversify risk by holding a portfolio of other investments. This unrestricted risk sharing lowers the cost of bearing risk. The efficiency in decision management is achieved by entrusting the management of the firm to managers who have the specific knowledge and technical skills to manage the firm. In open corporations most owners have no direct role in decision management.

Second, the cost of organizing firms as open corporations differs from proprietorships. The separation of ownership and management functions in open corporations leads to agency problems between owners and managers (see Jensen and Meckling (1986)). If both the owners and managers are utility maximizers, there is good reason to expect that managers will not always act in the best interest of the owners. As a consequence, there are conflicts of interests between owners and managers. This problem is controlled by writing and administering contracts between owners and managers. The cost of contracting is commonly called agency costs. [15] On the other hand, agency costs are avoided in proprietorships because the owner of the firm is also the decision maker.

Third, the decision rules for open corporations and proprietorships are different. Fama and Jensen (1985) discuss the investment rules for various organizational forms that are distinguished by the characteristics of their ownership structure. Their analysis indicates that investment decisions of open corporations can be modeled by the value maximization rule. [16] However, decisions of proprietorships cannot in general be modeled by the market value rule. An appropriate decision rule for proprietorships is to maximize the expected utility of wealth of the owner. [17] The optimal decisions under different decision rules vary and different organizational forms will make different decisions.

The benefits of specialization in risk-bearing and management functions, the agency cost of organizing firms as open corporations, and the different decision rules for open corporations and proprietorships are important issues that must be considered in extending the price-service equilibrium model. This could lead to empirically testable propositions of when one organizational form is expected to dominate others. For example, what implications does the size of different price-service market segments have on the competition among organizational forms for survival.

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[15] Jensen and Meckling (1976) define agency costs as the sum of the cost of monitoring activities by the owners, the bonding cost by managers and the cost due to divergence between the managers decisions and the decisions which would maximize the owners welfare.

[16] The Sharpe (1964)-Lintner (1965) capital asset pricing model can be used to determine the market value of an investment. The relevant measure of risk in their model is the nondiversifiable risk associated with the fluctuations in the economy.

[17] Fama and Jensen (1985) argue that if proprietors are active participants in the capital markets, then they will evaluate investment opportunities according to the market value rule.

Probability of product availability is one of the many dimensions of service on which consumers differentiate firms. Other dimensions of service include the variety of styles and brands available, the driving distance and the time it takes to reach the store, the layout of the store, the ease of finding the product, and the waiting time for service. [18] Future research should consider extending the model in this chapter to include the other dimensions of service.

[18] See Baumol and Ide (1957) and De Vany (1976).

CHAPTER 4.

Financial Justification of New Technologies

4.1. Introduction

Manufacturing processes in discrete parts manufacturing and equipment assembly are undergoing dramatic and rapid changes due to the introduction of new technologies involving flexible automation, robotics, automated material storage and handling, and the computer integration of manufacturing systems. Compared with conventional technologies, new technologies often cost more to acquire and install but have lower variable costs, offer better quality, reduced lead times, greater flexibility, and allow for improved manufacturing control. The procedures extant in many firms for evaluating capital investments in manufacturing processes appear to have been in place for over 20 years. They are typically oriented towards identifying cost savings arising from higher production rates and reduced labor costs, and justifying the initial investment in equipment by comparing it against the discounted present value of the cost reductions using some management specified discount factor or "hurdle rate". Seemingly, many new technologies either cannot be justified purely on these grounds, or appear to provide marginal benefits. [1]

The apparent inability of traditional modes of financial analysis like discounted cash flow to justify investments in new technologies has led a growing number of observers to propose abondoning such criteria for evaluating investments in new technologies. For example, Gold (1983) states: "One must keep remembering that in buying long-lasting facilities and equipment which embody major technological innovation, the fundamental objective is not to maximize net present value but rather to improve and safegaurd profitability over an extended period." He suggests that in evaluating investments in new technologies the net cash flows over a period of 8-10 years should be estimated without discounting. Hayes and Garvin (1982) attribute the use of discounted cash flow analysis by firms to the slower growth rate of capital investment and R&D spending in U.S.. They say that " ....the willingness of managers to view the future through the reversed telescope of discounted cash flow analysis is seriously shortcharging the futures of their companies." They also believe that " Beyond all else, capital investment represents an act of faith...."

Logue and West (1983) and Kaplan (1986) take issues with the detractors of discounted cash flow (DCF) analysis, and argue that it is unlikely that the theory of discounting future cash flows is either faulty or unimportant. Kaplan rightly states that "....the trouble must not lie in some unbreachable gulf between the logic of DCF and the nature of computer-integrated manufacturing (CIM) but in the poor application of DCF to these investment proposals. Managers need not -and should not- abandon the effort to justify CIM on financial grounds." Pinches (1982), Myers (1984) and Kaplan (1986) question whether capital bud-

[1] For example, see Baker(1984), Shewchuk (1984), Huber (1985), Stauffer (1986), Uteht (1986), Dornan (1987) and Sprow (1987).

geting procedures are being used wisely, but the criticisims are about how DCF is implemented in firm rather than the theory of discounting.

There are two major reasons for taking issue with the existing capital budgeting procedures. First, existing procedures focus too narrowly on easily quantifiable savings in labor, material and energy, but ignore the many additional benefits -reduced inventory, better quality, greater flexibility, shorter leadtimes, reduced throughput times, and less floor space- that are provided by new technologies. Some of these benefits such as inventory savings, less floor space, and better quality can be easily quantified and should be reflected in capital budgeting procedures. However, other benefits such as greater flexibility, shorter lead times, and reduced throughput times are harder to quantify. These benefits sometimes result in competitive advantages and/or market share benefits that are hard, if not impossible, to assess ex ante. The papers by Ayres and Miller(1981), Gerwin(1982), Gold(1982), Thompson and Paris(1982), Kaplan(1983), Jelinek and Goldhar (1984), and Meredith (1987a, 1987b) provide a useful discussion of some of the additional strategic benefits besides direct cost savings, that can be obtained from new technologies.

The other issue of concern is the discount rate used to evaluate investments in new technologies. The discount rate should reflect the riskiness of the investment and should be based on the opportunity cost of capital, that is, the rate of return available in the capital markets for investments of the same risk. Existing capital budgeting procedures typically use "hurdle rate" techniques which do not allow for variation in discount rate to account for the variation in riskiness of new technologies. Technologies with widely different risk characteristics are evaluated using the same discount rate.

The objective of this chapter is to address the issue of business risk of investment decisions in new technologies. Unlike present value procedures that use fixed discount rates, this chapter calculates the value of investments in new technologies varying the discount rates so that it accounts for the risk of the technology. Although there are many factors that affect the risk of the firm, this chapter focuses on the effect of the cost structure of new technologies on the risk of the firm.

The issue of cost structure of technology choice has been treated in the finance and accounting literature in terms of the "operating leverage" of a firm and its impact on risk. Operating leverage is defined as the ratio of variable profits (revenue minus variable costs) to operating profits (variable profits minus fixed operating costs). Rubinstein(1973), Brenner and Schmidt(1978), and Gahlon and Gentry(1982) demonstrate the relation between operating leverage and the risk of the firm. Lev(1974) provides empirical evidence that operating leverage is one of the determinants of the risk of the firm and that firms with higher operating leverage are more risky. However, the connection between specific technologies, such as robotics or flexible manufacturing systems, and their cost structure on the risk of the firm has not been made.

Section 4.2 presents evidence on the cost structure of new and conventional technologies. Examples from the literature are used to show that the cost structure of new technologies is different from that of conventional technologies. New technologies require a higher initial investment than conventional technologies, but have lower fixed operating costs per period (excluding depreciation) and lower variable costs per unit when compared to conventional technologies. This has an interesting implications for the riskiness of the future cash flows of the firm. Since both the fixed operating costs and variable costs are lower with new technologies, the future cash flows of the firm are less risky with new technologies than with conventional technologies. Therefore, the cash flows from new technologies should be discounted at a lower discount rate to determine the present value of the future cash flows, which should then be compared with the initial investment to make the investment decisions.

Section 4.3 uses the Capital Asset Pricing Model (CAPM), developed independently by Sharpe (1964) and Lintner (1965), to develop a model for computing the appropriate discount rate for evaluating technologies with different fixed and variable costs. In deriving the model it is assumed that the primary source of uncertainty is demand variability. I show that the discount rate is an increasing function of the breakeven point, that is, the ratio of fixed operating costs to contribution margin. Section 4.4 uses an example from a case study dealing with the evaluation of a flexible manufacturing system (FMS) to illustrate how the cost structure of the technology affects the discount rate. Section 4.5 shows that a different discount rate should be used when technologies are evaluated solely on the basis of costs than when they are evaluated on the basis of both revenues and costs.

4.2. Comparison of the Cost Structure of New and Conventional Technologies.

This section presents evidence to establish that new technologies have cost structures different from conventional technologies. Many examples of successful implementation of new technologies have appeared in the literature. These examples provide information on the investment required to replace conventional technologies with new technologies, and the savings in operating costs from investments in new technologies. Some of these examples are used to show that the cost structure of new and conventional technologies is different.

Sloggy (1984) describes the flexible manufacturing system (FMS) for machining locomotive parts (motor frames housing) at one U.S. manufacturer. Table 4.1 compares the operating performance and operating costs of the FMS system with the conventional stand-alone machines it replaced.

Table 4.1

Comparison of Investment and Operating Costs of Stand-alone Machines and a Flexible Manufacturing System at One U.S. Manufacturer. *

Comparison	Stand-alone Machines	Flexible Manufacturing System
Number of machines	29	9
Floor space	100,000 ft^2	20,000 ft^2
Typical number of machine loadings per part	10-11	4-5
Average in-process time for a part	16 days	16 hours
Average in-process inventory	$ 2.1 million	$ 80,000
Number of production workers	112	10
Total labor cost	$ 3.8 million	$ 0.34 million
Maintenance cost	$ 145,000	$ 27,000
Power consumption	$ 87,000	$ 27,000
Factory annual operating cost	$ 500,000	$ 100,000
In-process inventory cost at 10%	$ 210,000	$ 8,000
Total annual operating costs	$ 4.75 million	$ 0.5 million
Incremental investment	0	$ 16 million

* Source: Sloggy (1984).

The FMS has one third as many machine tools as the system it replaced. Twenty-nine manually operated machines were replaced by nine automated machining centers. As a result, floor space requirements were reduced by 80%, the typical number of times a part had to be loaded on separate machine was reduced by 50%, the average in-process time was reduced from 16 days to 16 hours, and the average in-process inventory was reduced from $2.1 million to $ 80,000. The capacity of the system increased from 4100 parts per year to 5600 parts per year, an increase of nearly 37%. The total number of production people required to support the machining activity over two shifts (operators, material handlers, inspectors, and supervisors) were reduced from 112 to 10. In addition to these benefits the new system improved part quality.

The impact on operating costs as a result of installing the new system is also dramatic. Total annual operating costs were reduced from $ 4.75 million to $ 0.5 million, a reduction of nearly 90%. The breakup of the labor costs into fixed and variable components is not available. Assuming that only part of the total labor costs is variable, the installation of the FMS reduced both fixed and variable operating costs. The total investment for the new system was $ 16 million. The net investment was only $ 14 million because of the one time reduction in inventory of about $ 2 million.

Hartley (1983) describes the flexible manufacturing system at Yamazaki's new factory at Minokamo in Japan. The new factory has a potential output of 120 lathes and machining centers per month. The FMS handles 550 part types per month and produces 11000 pieces per month. A comparison of the FMS with the conventional technology is shown in Table 4.2. As the table shows, the installation of the FMS has resulted in fewer machines, reduced floor space, reduced processing time, and reduced the number of personnel from 195 to 39, a reduction of 80%. The investment in this FMS is £ 40 million.

Table 4.2

Yamazaki's Comparison for Minokamo Flexible Manufacturing System with the Conventional System.*

Comparison	Conventional System	Flexible Manufacturing System
Number of machines	90	43
Floor space	16,500 m^2	6,600 m^2
Process time		
Machining time	35 days	3 days
Unit assembly	14 days	7 days
Overall assembly	42 days	20 days
Total processing time	91 days	30 days
Number of operators		
Factory	170	36
Production control	25	3
Total Operators	195	39

* Source: Hartley (1983).

Detailed cost information on this FMS is not available. The operating performance of the FMS suggests lower operating costs when compared to conventional technology. The benefits of reduced floor space and reduced processing time together with the substantial savings in labor costs should result in lower fixed and variable manufacturing costs. [2]

Another flexible manufacturing system at Yamazaki is described in

[2] Jaikumar (1986) describes the performance of one Japanese factory before and after the introduction of flexible automation. His figures, except for the number of parts produced per month and the types of parts produced per month, are the same as shown in Table 4.2. He does not mention the name of the company.

Hollingum (1983).[3] This FMS, which is in operation since 1981, produces about 1400 workpieces a month of 74 different part types for building machine tools. The FMS is designed for 24 hour continuous operation, unattended in the third shift. Table 4.3 compares the FMS with a conventional machine shop.

Table 4.3

Yamazaki's Comparison of a Flexible Manufacturing System with a Conventional Machine Shop.*

Comparison	Conventional Machine Shop	Flexible Manufacturing System
Number of machines	68	18
Floor space	70,000 ft^2	30,000 ft^2
Number of production workers	215	12
In-process Time	90 days	3 days
Annual labor cost	$ 4.0 million	$ 227,000
In-process Inventory	$ 5.0 million	$ 218,000
Capital cost (incl. land and building	$ 14.0 million	$ 18.0 million

* Source: Hollingum (1983).

The operating performance of this FMS is truly dramatic: a reduction in number of machines by 74%, in floor space by 70%, in processing time by 97%, and in number of employees by 94%. A one time inventory saving of $ 4.8 million was realized with the FMS. Furthermore, the operating costs with the FMS are much lower compared to the conventional machine shop, as evident from the

[3] Hollingum also summarizes the benefits and savings realized by ten other flexible manufacturing systems installed by various Japanese firms. His summary is based on the findings and conclusions of two group of observers from Britain and other European countries, who visited Japan to study flexible manufacturing systems in that country.

annual labor cost. The capital investment in this FMS is 30% higher than the capital investment in the conventional machine shop. [4]

Jelinek and Goldhar (1984) discuss the benefits from a highly advanced flexible manufacturing system at Messerschmitt-Bolkow-Blohm in Augsburg, West Germany. The FMS has been in full operation since 1980 and machines titanium and other material components for the Tornado fighter aircraft. The total system costs about $ 50 million. The FMS system has reduced in-process times by 26%, the number of machines by 44%, floor space requirements by 39%, personnel by 44%, and overall annual operating costs by 24%. This system is utilized 75 to 80 percent of the time, in contrast to typical stand-alone machines that are only utilized 15 to 30 percent of the time. Jelinek and Goldhar do not provide detailed cost information of the FMS. However, from the information given above, one would expect that the operating costs with the FMS will be lower than those with the conventional stand-alone machines.

The experience with a flexible manufacturing system of one U.S. manufacturer of air-handling equipment is described in Kaplan (1986). Compared with conventional technology, the FMS technology increased utilization from 30-40% to 80-90%, reduced scrap and rework by $ 60,000 annually, reduced inventory from $ 2 million to $ 1.1 million, and reduced the number of employees (including indirect workers) from 52 to 14. As a result of the 38 fewer employees the labor costs were reduced by $ 1.4 million. The incremental investment for replacing the conventional technology with the FMS was $ 9.2 million. The FMS also offered unlimited flexibility to modify component mix, and offered better component quality.

[4] This example has also appeared in Bylinski (1983), Baker (1984), Shewchuk (1984) and Kaplan (1986).

The next example is from a case study in which Phillip Lederer of the University of Rochester and I were involved. This case study was concerned with the evaluation of a flexible manufacturing system in a department of one U.S. manufacturer. The department manufactures sheet metal parts for industrial and consumer products. Currently, conventional technology -consisting of a number of stand-alone machines- is used to manufacture sheet metal parts. The department is considering the replacement of conventional technology with a flexible manufacturing system, which will use state-of-the-art technologies such as laser metal cutting, robotics, automated storage and retrieval sytems, and direct computer control of fabrication equipment. Table 4.4 compares the cost structure of the conventional technology with the FMS. For confidentiality reasons Table 4.4 contains disguised data, obtained by scaling the actual data by a common factor.

In Table 4.4 fixed overhead costs include indirect labor, material handling, maintenance, software engineers, and indirect material and supplies. Perhaps, the most interesting observation is that both the fixed operating costs and the variable costs per unit with the FMS are lower than those with the conventional technology. The variable costs per part are reduced from \$ 3.68 to \$ 2.4, a reduction of 35%, and the fixed operating costs per year are reduced from \$ 3.76 million to \$ 2.28 million, a reduction of 39%. Clearly, the cost structure of the FMS is different from the conventional technology. The FMS also provided additional benefits such as reduced inventory, flexibility, and reduced lead times. The incremental investment for the FMS is \$ 7.5 million. [5]

Frost & Sullivan, Inc. (New York) in a recent study of 20 U.S. systems indicates that switching to flexible manufacturing systems from other methods of

[5] In section 4.3, I use this case study to illustrate how the fixed and variable costs of a technology affects the discount rate.

Table 4.4

Comparison of the Cost Structure of a Conventional Technology with a Flexible Manufacturing System at One U.S. Firm.

Comparison	Conventional Technology	Flexible Manufacturing System
Number of direct workers	35	21
Average in-process time for a part	9 weeks	3 days
Average in-process inventory	$ 260,000	$ 35,000
Finished goods inventory	$ 818,000	$204,000
Number of part types	3,000	3,000
Average number of pieces produced/year	544,000	544,000
Variable Labor cost/part	$ 2.15	$ 1.30
Variable material cost/part	$ 1.53	$ 1.1
Total variable cost/part	$ 3.68	$ 2.40
Annual overhead costs	$ 3.15 million	$ 1.95 million
Annual tooling costs	$ 470,000	$ 300,000
Annual inventory costs	$ 141,000	$ 31,500
Total annual fixed operating costs	$ 3.76 million	$ 2.28 million
Incremental investment	0	$ 7.5 million

manufacturing have resulted in substantial benefits. [6] Table 4.5 summarizes these benefits. Their study shows that switching to FMS has resulted in a reduction in direct labor of 50 – 80%, a reduction in number of machines of 60 – 90%, a reduction in floor space of 30 – 80%, a reduction in processing times of 30 – 90%, a reduction in product costs of 25 – 75%, plus other benefits such as reduction in number of operations and setups, and an increase in machine efficiency.

[6] The results of this study are described in Palframan (1987).

Table 4.5

Benefits of Flexible Manufacturing Systems.*

Comparison	Prior Method	Flexible Manufacturing System	Average Improvement	Range of Improvement For Total Sample[1]
Number of machines	29	9	70%	60 – 90%
Floor space	1500 m^2	500 m^2	66%	30 – 80%
Direct labor	70	16	77%	50 – 88%
Product cost	$2000	$1000	50%	25 – 75%
In-process Time	18.6 days	4.2 days	77%	30 – 90%
Number of operations	15	8	47%	
Number of setups	13	5	62%	10 – 75%
Machine efficiency	20%	70%	50%	15 – 90%

* Source: Frost & Sullivan, Inc.(New York).
[1] Based on a sample of 20 U.S. operating systems.

Jaikumar (1986) has done a detailed study of 95 flexible manufacturing systems in United States and Japan. Table 4.6 gives his comparison of the manpower requirements of various manufacturing systems for metal-cutting operations in one industry. If it took 100 people in a conventional Japanese factory to make a certain number of parts, it would take 194 people in a conventional U.S. factory, but only 43 in a Japanese FMS equipped factory. If U.S. firms could achieve the same workforce level as Japanese FMS, they would reduce manpower in manufacturing overhead by 92%, in fabrication by 88%, in assembly by 54%, and in engineering by 53%. These dramatic savings in manpower requirements would reduce both the fixed manufacturing overhead and the variable manufacturing costs.

There are many other examples of successful applications of new technologies

Table 4.6

Comparison of Manpower Requirements of Conventional Technology with Flexible Manufacturing Systems for Metal-Cutting Operations.*

Comparison	Conventional Systems United States	Conventional Systems Japan	Flexible Manufacturing Systems Japan
Engineering	34	18	16
Manufacturing Overhead	64	22	5
Fabrication	52	28	6
Assembly	44	32	16
Total number of workers	194	100	43

* Source: Jaikumar(1986).

in manufacturing firms. For example, see the papers by Bylinsky (1983,1986), Baker (1984), Hundy (1984), Jelinek and Goldhar (1984), Primrose and Leonard (1984), Shewchuk (1984), Ashburn and Jablonowski (1985), Miller (1985), Saporito (1986), and Sprow (1987). These papers underscore the dramatic reduction in labor costs, number of machines, floor space, inventory, scrap and rework, and lead times when firms acquire new technologies.

The case-style evidence presented in this section shows that the cost structure of new technologies is different from that of conventional technologies. One way to describe the differences in the cost structure of new and conventional technologies is to specify the cost structure of a technology in terms of three attributes: (1) the initial investment, (2) the fixed operating cost per period, and (3) the variable cost per unit. The evidence suggests that new technologies require a higher initial investment but have lower variable costs per unit when compared to conventional technology. This result is what would be expected, for example, in technologies as they become more automated and less labor intensive. What is perhaps surprising is that the evidence suggests that new technologies have lower fixed operating costs per period when compared to conventional technology. This is because of the reduction in number of machines, number of operations, number of setups, the number of supporting staff, and floor space that firms achieve when they acquire new technologies. This result must be interpreted with caution as it is based on a small sample and may not be true for all firms. The implications of the differences in the cost structure of technologies on the discount rate for evaluating these technologies are discussed in the next section.

4.3. A Model for Computing the Discount Rate

This section presents a model that captures the relation between the cost structure of technologies and the appropriate discount rate for evaluating technologies. The following assumptions are made in the basic model formulation.

A1. The firm manufactures and sells a single product at a fixed price P. The demand, $\tilde{D}$, for the product is stochastic with mean $\bar{D}$, and variance σ_D^2.

A2. The firm exists for a single period. At the beginning of the period the firm selects its technology. The demand uncertainty is resolved at the end of the period when the firm observes demand. Production is instantaneous and the quantity produced equals the quantity demanded. The firm liquidates itself at the end of the period. At liquidation the salvage value of the technology is zero.

A3. The choice of technology has no impact on the demand and the selling price.

A4. The cost structure of the technology chosen by the firm can be characterized by three parameters: the initial investment, I, the fixed operating costs per period, F, and the variable costs per unit, C.

A5. The initial investment is incurred at the beginning of the period, the fixed operating costs and the variable costs per unit are incurred at the end of the period, and the revenue is realized at the end of the period.

A6. The firm is an all-equity financed firm, where the equity holders contribute the initial investment, I.

A7. All taxes are zero.

A8. The objective is to determine the appropriate discount rate for computing the net present value (NPV) of the equity holders' claims.

As of the beginning of the period, the firm's end-of-period cash flows, $\tilde{X}$, are uncertain and can be written as:

$$\tilde{X} = (P - C)\tilde{D} - F. \tag{4.1}$$

The net present value (NPV) of the technology chosen by the firm is the present value of the firm, $V(\tilde{X})$, less the beginning-of-period cash outlay, I. Note that the value of the firm equals the present value of the uncertain cash flow, $\tilde{X}$.

Conventional capital budgeting techniques use some form of discounted cash flow analysis to calculate the present value of an investment. Using the simplest form of the present value formula, the present value of $\tilde{X}$ is:

$$V(\tilde{X}) = \frac{(P - C)\bar{D} - F}{(1 + R)}, \tag{4.2}$$

where $V(\tilde{X})$ is the present market value of $\tilde{X}$; $((P - C)\bar{D} - F)$ is the expected value of $\tilde{X}$; and R is the discount rate or the opportunity cost of capital.

The appropriate discount rate for evaluating an investment is defined as the equilibrium expected rate of return on securities equivalent in risk to the investment being valued. Hence, to determine the discount rate we need to answer two questions. First, how is risk defined? And second, what is the relation between risk and the equilibrium expected rate of return.

Developments in modern finance theory have provided managers with a methodology for answering these two questions. The theory underlying this methodology is embodied in the model called the capital asset pricing model (CAPM), developed independently by Sharpe (1964) and Lintner (1965). CAPM is a theroetical representation of how financial assets, such as bonds and stocks, are valued in the capital markets. CAPM can be used to determine the appropriate discount rate for evaluating an investment. It defines risk, shows how the risk can be measured, and it provides a relation between risk and discount rate.

Although the total risk of an individual security is measured by the variance of its return, the relevant measure of risk in pricing an individual security is the nondiversifiable risk of the security. Investors in capital markets hold well diversified portfolios of securities and can diversify away part of the total risk of a security by portfolio formation. The risk that cannot be diversified away is called the nondiversifiable risk of a security, and is measured by the covariance of its return with the return on the market portfolio of all assets. This risk measure is commonly called a security's nondiversifiable risk. CAPM gives the relation between the nondiversifiable risk of a security and its expected return. In the simplest form of CAPM the following expression gives the risk/expected return relation on security j:

$$E(\tilde{R}_j) = R_f + \beta_j(E(\tilde{R}_m) - R_f), \tag{4.3}$$

where $E(\tilde{R}_j)$ is the expected return on security j; R_f is the riskless rate of return;

$E(\tilde{R}_m)$ is the expected return on the market portfolio of all assets; and $\beta_j = \frac{\mathrm{Cov}(\tilde{R}_j, \tilde{R}_m)}{\sigma_m^2}$, the covariance between the return on security j and the market return divided by the variance of the market return, is the measure of the relative risk of the security j, commonly referred to as the security's "beta". [7]

CAPM can be used to determine the discount rate for evaluating an investment. If we know the beta of an investment then the expected rate of return on securities equivalent in risk to the investment being valued can be computed from equation (4.3). By definition, this expected rate of return is the appropriate discount rate for evaluating the investment. Note from equation (4.3) that the higher the beta of an investment, the higher is the discount rate.

Next consider the problem of determining the appropriate discount rate for computing the value of the firm in our technology choice model. Recall that the value of the firm is the present value of the risky cash flow, $\tilde{X}$, given in equation (4.1). If the beta of this cash flow is known, then the discount rate can be computed from equation (4.3). Appendix C uses the certainty equivalent form of the CAPM (see equation (C.2) in Appendix C) and the definition of beta, $\beta = \frac{\mathrm{Cov}(\tilde{R}, \tilde{R}_m)}{\sigma_m^2}$, to show that the beta of the firm can be expressed as:

$$\beta = \frac{\beta_D}{1 - \frac{F}{(P-C)(\bar{D}-\lambda \mathrm{Cov}(\tilde{D}, \tilde{R}_m))}}, \tag{4.4}$$

where β_D is defined as the beta of demand, and is given by

$$\beta_D = \frac{(1+R_f)\mathrm{Cov}(\tilde{D}, \tilde{R}_m)}{\sigma_m^2(\bar{D} - \lambda \mathrm{Cov}(\tilde{D}, \tilde{R}_m))}, \tag{4.5}$$

and R_f is the risk-free rate of return; $\bar{D}$ is the expected demand; $\mathrm{Cov}(\tilde{D}, \tilde{R}_m)$ is the covariance of demand with the market return; and $\lambda = [E(\tilde{R}_m) - R_f]/\sigma_m^2$,

[7] See Brealey and Myers (1981, chapters 4-7), and Mullins (1982), for an introductory exposition of the capital asset pricing model.

the expected risk premium on the market divided by the variance of the market return, is the market price per unit of risk. The term $(\bar{D} - \lambda \mathrm{Cov}(\tilde{D}, \tilde{R}_m))$ is called the certainty equivalent of demand.

The beta of demand measures the risk of demand and can be interpreted in the following way. Suppose there exists a firm that uses a technology where fixed costs are zero and all costs are variable. Such a firm will face the risk of demand. The beta of this firm is the beta of demand.

Equation (4.4) shows that for given values of beta of demand and the certainty equivalent of demand, $(\bar{D} - \lambda \mathrm{Cov}(\tilde{D}, \tilde{R}_m))$, the beta of the firm is an increasing function of the the ratio of fixed operating costs, F, to the contribution margin per unit, $(P - C)$. This ratio is called a technology's "break-even" point. Other things being equal, the technology with the higher breakeven point will have the higher beta. [8]

Next, consider the implications of our model for evaluating investments in new and conventional technologies. The case style evidence presented in section 4.2 shows that the fixed operating costs per period and the variable costs per unit with new technologies are different from those with conventional technologies. Hence, different discount rates must be used to evaluate these technologies. More

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[8] In equation (4.4), the term $1/(1 - \frac{F}{(P-C)(\bar{D}-\lambda \mathrm{Cov}(\tilde{D}, \tilde{R}_m))})$, which can be written as $\frac{(P-C)(\bar{D}-\lambda \mathrm{Cov}(\tilde{D}, \tilde{R}_m))}{(P-C)(\bar{D}-\lambda \mathrm{Cov}(\tilde{D}, \tilde{R}_m))-F}$, measures the degree of operating leverage of the technology. This measure is slightly different from the definition of operating leverage in finance and accounting literature. This literature defines the degree of operating leverage as $\frac{(P-C)D}{(P-C)D-F}$, where D is the number of units sold (for example, see Brenner and Schmidt (1978), Gahlon and Gentry (1982), and Mandelkar and Rhee (1984)). The only difference between this measure and ours is that we use the certainty equivalent of demand as the measure of demand. In both measures the degree of operating leverage is an increasing function of the breakeven point.

specifically, our evidence suggests that new technologies have lower fixed operating costs per period and lower variable costs per unit than conventional technologies. If we assume that the choice of technology has no impact on the demand or the selling price, then the breakeven point of new technologies is lower than that of conventional technologies. This implies that the beta of new technologies is lower than that of conventional technologies. Therefore, the future cash flows from new technologies should be discounted at a lower discount rate to determine the present value of the cash flows, which should then be compared with the initial outlay to make the investment decision.

It must be stressed that a lower discount rate should be used for evaluating new technologies if new technologies have lower fixed and lower variable costs than conventional technologies. There can be cases where new technologies have higher fixed operating costs but lower variable costs per unit than conventional technology. In such cases, it is not necessarily true that a lower discount rate should be used to evaluate new technologies. The higher fixed costs together with the lower variable costs could result in a higher breakeven point for the technology which would imply a higher beta, and therefore, a higher discount rate. [9]

4.4. An Example

Having argued that differences in the cost structure of new and conventional technologies imply different discount rates for evaluating these technologies, I now examine the important (and practical) issue of under what conditions the difference in the discount rates can be large. For this purpose, consider the example of

[9] Since new technologies are more automated and less labor intensive than conventional technologies, it is reasonable to expect that new technologies have lower variable costs than conventional technologies. Hence, in this chapter, the case where new technologies have higher variable costs than conventional technologies is not considered.

the cost structures of the conventional technology and the flexible manufacturing system (FMS) given earlier in Table 4.4. As mentioned earlier, these technologies are used for manufacturing sheet metal parts for industrial and consumer products. The average demand for the sheet metal parts is 544,000 pieces per year. Conventional technology has fixed operating costs of \$3.76 million per year and variable costs of \$3.68 per piece. The FMS has fixed operating costs of \$2.28 million per year and variable costs of \$2.4 per piece. The cost of acquiring and installing the FMS is \$7.5 million.

In computing the discount rate the average return during 1986 on new issues of Treasury bills with 90-days maturity, 6%, is used to measure the risk-free rate of return, R_f, (this rate is obtained from the Economic Report of the President to the U.S. Congress, January 1987). The market risk premium, $(E(\tilde{R}_m) - R_f)$, is estimated at 8.5%, and the standard deviation of the market return is estimated at 21% (these estimates are obtained from Ibbotson and Sinquefield (1985)).[10] Using these estimates of the market risk premium and the standard deviation of the market return the market price per unit of risk is estimated at 1.9 percent per unit of variance.

Table 4.7 gives the discount rates for the conventional technology and the FMS for different values of the beta of demand and for three different values of the selling price per piece. [11] Equations (4.3), (4.4), and (4.5) are used to compute

[10] Ibbotson and Sinquefield's estimate of the market risk premium is the arithmetic average of the market risk premium over the years 1926-1984. Merton (1980) observes that estimating the market risk premium using the average over such long a time period fails to account for the effect of changes in the level of market risk. The level of market risk may not be stationary over that long a period. Confronting this problem is beyond the scope of this chapter. However, Merton's estimates of the market risk premium using models that account for the changing variance of the market and using data over the years 1926-1978, vary from 8.2% to 12.0%.

[11] The selling price of \$13.5 per piece is the actual average selling price per piece

the discount rates. For a given value of the beta of demand, the implicit value of the covariance of demand with the market return is computed from equation (4.5). The beta of the technology is then computed from equation (4.4). Finally, equation (4.3) is used to determine the discount rate for the beta obtained from (4.4). The table also gives the break-even point for the two technologies, expressed as a percentage of the average demand.

Table 4.7

Comparison of the Discount Rates for Evaluating the Conventional Technology and the Flexible Manufacturing System (FMS) for Various Values of Beta of Demand and Selling Prices.

	Price= $12.0		Price= $13.5		Price= $15.0	
Beta of Demand	Conv. Tech.	FMS	Conv. Tech.	FMS	Conv. Tech.	FMS
0.1	11.2%	7.5%	8.9%	7.3%	8.2%	7.3%
0.2	17.0%	9.0%	12.0%	8.7%	10.5%	8.5%
0.3	23.0%	10.5%	15.1%	10.1%	12.8%	9.8%
0.4	30.0%	12.0%	18.4%	11.4%	15.2%	11.1%
0.5	37.5%	13.5%	22.0%	12.8%	17.7%	12.4%
Breakeven Point*	83.0%	44.0%	70.0%	38.0%	61.0%	33.0%

* Breakeven point is expressed as a percentage of the average demand 544,000 pieces per year.
Conventional technology has fixed operating costs of of $3.76 million per year and variable costs of $3.68 per piece. The FMS has fixed operating costs of $2.28 million per year and variable costs of $2.4 per piece.

The main observation about the discount rates in Table 4.7 is that the difference in the discount rates for evaluating the two technologies is largest when the selling price is low and the beta of demand is high. The reason being that

multiplied by the constant used to disguise the data in Table 4.4.

the breakeven points of the two technologies are very different. The breakeven point of the conventional technology is very high and slightly less than twice the breakeven point of the FMS. Furthermore, when the breakeven point is high and the beta of demand is high, the discount rate is more sensitive to changes in the breakeven point. For example, when beta of demand is held constant at 0.5, an increase in the breakeven point from 33% to 44% increases the discount rate by about 1.0%, whereas an increase in the breakeven point from 61% to 70% increases the discount rate by about 4.3%. On the other hand, when the beta of demand is held constant at 0.1, the corresponding increases in the discount rates are 0.2% and 0.7%. Since the breakeven point of the conventional technology is much higher than that of the FMS, a decrease in price and/or an increases in the beta of demand causes a larger increase in its the discount rate than that of the FMS. Hence, the difference in the discount rates is larger when the price is low and beta of demand is high.

Clearly, it is the difference in the breakeven points of the two technologies which is causing the large difference in the discount rates. When one technology has a high breakeven point compared to the other, the difference in the discount rates of the two technologies can be large. In such situations different discount rates must be used to determine the net present value of different technologies.

To illustrate the impact of using different discount rates on the present value calculations, consider again the cost structure of the conventional technology and the flexible manufacturing system (FMS) given earlier in Table 4.4. Suppose the present value of the net cash flows from the FMS are calculated in two ways. First, by using the appropriate discount rate from our model, that is, the discount rate adjusted for the cost structure of the FMS. Second, by using the discount rate for the conventional technology from our model to discount the net cash flows from the FMS, that is, the discount rate is not adjusted for the cost structure of the FMS. For illustrative purposes assume the following: (1) Selling price is \$13.5 per unit, (2) the FMS technology will last for one year, and (3) the present value (PV) of the net cash flows from the FMS is given by

$$PV = \frac{E(\tilde{X})}{(1+R)},$$

where $E(\tilde{X})$ is the expected net cash flow from the FMS, and R is the discount rate. Given these assumptions and the data in Table 4.4, the expected net cash flow from the FMS is \$3.75 million per year. [12]

Table 4.8 gives the present values of the net cash flows from the FMS. Columns 2 and 4 give the discount rates for the FMS and the conventional technology, respectively, when price is \$13.5 per unit (see Table 4.7). The present values of the net cash flows from the FMS using the discount rates in columns 2 and 4 are given in columns 3 and 5, respectively. The last column gives the underestimation (in percentage) of the present value of the FMS when the conventional technology's discount rate is used to discount the net cash flows from the FMS, that is, the difference in the present values of columns 3 and 5 divided by the present value of column 3.

Table 4.8 shows that when the discount rates are not adjusted for the cost structure of the FMS, the underestimation of the present values of the net cash flows from the FMS increases as the difference in the discount rates of the two technologies increases. When this difference is 1.6% (beta of demand = 0.1),

[12] Recall that the FMS has fixed operating costs of \$2.28 million per year and variable costs of \$2.4 per piece. The average demand is 544000 units per year. Since the price is assumed as \$13.5 per piece, the expected net cash flows are $(13.5 - 2.4) * .544 - 2.28 = \$3.75$ million.

Table 4.8

Present Value (PV) of the Net Cash Flows from the FMS When (a) the Discount Rates are Adjusted for the Cost Structure of the FMS, and (b) When the Discount Rates are Not Adjusted for the Cost Structure of the FMS and are Based on the Cost Structure of the Conventional Technology.

Beta of Demand (1)	Discount Rate For FMS (2)	Correct Estimate of Present Value of FMS* (3)	Discount Rate For Conv. Tech. (4)	Incorrect Estimate of Present Value of FMS* (5)	Underestimation in Present Value of FMS (6)= [(3)-(5)]/(3)
0.1	7.3%	3.50	8.9%	3.45	1.4%
0.2	8.7%	3.45	12.0%	3.35	2.9%
0.3	10.1%	3.41	15.1%	3.26	4.4%
0.4	11.4%	3.37	18.4%	3.17	5.9%
0.5	12.8%	3.33	22.0%	3.08	7.5%

* Present values in millions of dollars. Present values computed under the following assumptions: (1) Selling price is $13.5 per piece, and (2) the FMS will last for one year.
Conventional technology has fixed operating costs of of $3.76 million per year and variable costs of $3.68 per piece. The FMS has fixed operating costs of $2.28 million per year and variable costs of $2.4 per piece.

using the discount rate of the conventional technology underestimates the present value of the net cash flows from the FMS by about 1.4%. The underestimation increases as the difference in discount rates increases and is about 7.5% when the difference in discount rate is 9.2% (beta of demand = 0.5). Note that the present values have been computed by considering the net cash flows from only one year. Because of discounting the underestimation (in percentage terms) will increase with an increase in the time period over which the present values are computed (typically 5 to 10 years). Over longer time periods the underestimation in the present values of the net cash flows from the FMS can be significant if the discount rates are not adjusted for the cost structure of the FMS. This could lead to the conclusion that the investment in the FMS is a negative net present value project which could be wrong.

4.5. Extensions

The model developed above considers both the revenues and the costs in determining the appropriate discount rate for evaluating investments in different technologies. In practice, firms often evaluate investments in new technologies solely on the basis of costs. [13] Two approaches are commonly used. The first approach is to determine the present value of total costs, which is the sum of the initial investment and the present value of all future costs. The decision criterion is to choose the technology that minimizes the present value of total costs. The other approach is to determine the present value of the cost savings and compare it with the incremental investment. The decision criterion is to invest in the new technology if the present value of the cost savings is greater than the incremental investment. Next, consider the extensions of the model to determine the appropriate discount rate for evaluating technologies solely on the basis of costs. I show that a different discount rate should be used when technologies are evaluated solely on the basis of costs than when both revenues and costs are

[13] There are at least two reasons for evaluating technologies solely on the basis of costs. First, the revenue implications of the many strategic benefits- better quality, reduced lead times, better customer response times, and flexibility- that are provided by new technologies are difficult, if not impossible, to estimate ex ante. Evaluating new technologies on the basis of costs gives a conservative estimate of the benefits, which can then be supplemented with the manager's subjective judgement on the value of the strategic benefits. Second, manufacturing has the responsibility of evaluating investments in new technologies. Manufacturing is usually organized as a cost center than as a profit center. Hence, it is appropriate to have procedures for evaluating technologies solely on the basis of costs.

considered.

Discount Rate for Evaluating Technologies on the basis of Total Costs

The assumptions in developing the model for computing the discount rate for evaluating technologies on the basis of total costs are the same as outlined at the beginning of this section, except that price is not relevant in this model. As of the beginning of the period, the firm's end-of-period total costs, $\tilde{Y}$, are uncertain and can be written as:

$$\tilde{Y} = C\tilde{D} + F. \tag{4.6}$$

It can be shown that the beta of the above cash flow can be expressed as

$$\beta = \frac{\beta_D}{1 + \frac{F}{C(\bar{D} - \lambda \mathrm{Cov}(\tilde{D}, \tilde{R}_m))}}, \tag{4.7}$$

where β_D is the beta of demand. [14]

It will be useful to compare equation (4.7) which gives the beta for evaluating technologies on the basis of total costs with equation (4.4) which gives the beta when both revenues and costs are considered. Observe that when both the fixed operating costs per period and the variable costs per unit are positive, the beta from equation (4.7) is lower than the beta of demand whereas the beta from equation (4.4) is greater that the beta of demand. This means that when both fixed and variable costs are positive, a lower discount rate should be used for evaluating technologies on the basis of total costs than when technologies are evaluated on the basis of revenues and costs. [15] Furthermore, when technologies are evaluated on the basis of total costs the lower bound on the beta is zero (when

[14] The derivation of equation (4.7) is similar to the derivation of equation (4.4). To derive equation (4.7) replace $(P - C)$ by C and F by $-F$ in equation $(C.1)$ of Appendix C, and follow the derivation in the appendix.

[15] Recall that discount rate is directly proportional to the beta.

variable costs are zero) and the upper bound on the beta is the beta of demand (when fixed costs are zero). On the other hand, when technologies are evaluated on the basis of both revenues and costs, the beta of demand is a lower bound on the beta (when fixed costs are zero). It is not possible to specify a meaningful upper bound in this case.

The intuition behind the model for computing the discount rate when total costs are considered is simple. If all costs are fixed, then the cash flows are not risky and the beta of the cash flows is zero. If all costs are variable, then the cash flows have the same risk as that of the demand and the beta of the cash flows equals the beta of demand. When some costs are fixed and some are variable, then the riskiness of the cash flows is a weighted average of the riskiness of the fixed and variable costs. The beta of the variable costs is weighted by the ratio of the value of the variable costs to the value of the total costs, and the beta of the fixed costs is weighted by the ratio of the value of fixed costs to the value of the total costs. Since the beta of fixed costs is zero, only the beta of demand and the weight assigned to this beta are relevant in computing the weighted average. When fixed costs are positive, the weight assigned to the beta of demand is less than 1. Therefore, the beta of total costs is less than the beta of demand.

Next consider the implications of the difference in the cost structure of conventional and new technologies on the appropriate discount rate for evaluating these technologies on the basis of total costs. Equation (4.7) shows that beta is a decreasing function of the ratio of fixed to variable costs. If new technologies have lower fixed operating costs per period and lower variable costs per unit than conventional technology, then it is difficult to say what happens to the discount rate for new technologies. This is because the ratio of fixed to variable costs with the new technologies can be lower or higher than that of the conventional technologies. This means that the discount rate for new technologies can be higher or lower than that of conventional technologies. But, when new technologies have higher fixed costs and lower variable costs than conventional technologies, the discount rate for evaluating new technologies should be lower than that of conventional technologies. The ratio of fixed to variable costs is higher with new technologies than with conventional technologies, and we know from equation (4.7) that beta is a decreasing function of this ratio.

Discount Rate for Evaluating Technologies on the Basis of Costs Savings.

An alternative method for evaluating technologies on the basis of costs is to determine the present value of the cost savings and compare this with the incremental investment. Next consider a model for computing the appropriate discount rate when technologies are evaluated on the basis of cost savings.

The assumptions of the model are the same as outlined at the beginning of this section. In addition, assume that the firm is currently operating with a conventional technology. Let F_c be the fixed operating costs per period and C_c be the variable costs per unit of the conventional technology. The firm is considering investing in a new technology. Let F_n be the fixed operating costs per period and C_n be the variable cost per unit of the new technology. If the firm invests in the new technology at the beginning of the period, the end-of-period cost savings, $\tilde{Z}$, are uncertain and can be written as:

$$\tilde{Z} = (C_c - C_n)\tilde{D} + (F_c - F_n). \tag{4.8}$$

It can be shown that the beta of the above cash flow can be expressed as

$$\beta = \frac{\beta_D}{1 + \frac{(F_c - F_n)}{(C_c - C_n)(\bar{D} - \lambda \mathrm{Cov}(\tilde{D}, \tilde{R}_m))}}, \tag{4.9}$$

where β_D is the beta of demand. [16]

Equation (4.9) shows that the beta of cost savings depends on the ratio of the savings in fixed costs $(F_c - F_n)$ to the savings in variable costs $(C_c - C_n)$. When this ratio is positive the beta of cost savings is less than the beta of demand. This ratio will be positve when new technologies have lower fixed operating costs and lower variable costs per unit than conventional technologies. But, when new technologies have higher fixed costs and lower variable costs than conventional technologies, the ratio of the savings in fixed costs to the savings in variable costs is negative, and the appropriate beta is greater than the beta of demand. Furthermore, if only variable costs are reduced then the beta of cost savings is the beta of demand. If only fixed costs are reduced, then the beta of the cost savings is zero. The intuition behind these results are similar to the intuition behind the results from the model that considers total costs.

The main point from the discussion on the appropriate discount rate for evaluating technologies solely on the basis of costs is that discount rates depend not only on the cost structure of the technology but also on the method used, that is, the total costs or the cost savings method. Furthermore, when only costs are considered the appropriate discount rates are different from the discount rate when both revenues and costs are considered. In many cases, when only costs are considered the appropriate discount rate can be lower than when both revenues and costs are considered.

[16] The derivation of equation (4.9) is similar to the derivation of equation (4.4). To derive equation (4.9) replace $(P - C)$ by $(C_c - C_n)$ and F by $-(F_c - F_n)$ in equation $(C.1)$ of Appendix C, and follow the derivation in the appendix.

Table 4.9 gives the appropriate discount rates for evaluating the conventional technology and the FMS, whose cost structures are given in Table 4.4, solely on the basis of costs. The discount rates for evaluating the conventional technology and the FMS on the basis of total costs are given in columns 2 and 3, respectively. The last column gives the discount rate for the cost savings if the conventional technology is replaced by the FMS.

Table 4.9

Discount Rates for Evaluating the Conventional Technology and Flexible Manufacturing System (FMS) on (a) the Basis of Total Costs and (b) the Basis of Cost Savings, for various Values of Beta of Demand.

	Discount Rates		
Beta of Demand 1	Total Costs Conv. Tech. 2	Total Costs FMS 3	Cost Savings 4
0.1	6.3%	6.3%	6.3%
0.2	6.5%	6.6%	6.5%
0.3	6.8%	6.9%	6.8%
0.4	7.1%	7.2%	7.0%
0.5	7.4%	7.5%	7.3%

Conventional technology has fixed operating costs of of $3.76 million per year and variable costs of $3.68 per piece. The FMS has fixed operating costs of $2.28 million per year and variable costs of $2.4 per piece.

Table 4.9 shows that for a given value of the beta of demand, the discount rate are nearly the same whether the technologies are evaluated on the basis of total costs or cost savings. This is because the ratio of fixed to variable costs of the two technologies is not significantly different. [17] The value of this ratio for the

[17] Recall that in the models based on costs the beta of the cash flows depends

conventional technology is 1.02 million ($3.76 million/ $3.68), and for the FMS it is 0.95 million ($2.28 million/ $2.4). If the conventional technology is replaced by the FMS, the ratio of the savings in fixed costs to the savings in variable costs is 1.16 million ($1.4 million/ $1.28). Even though this ratio is about 16% higher than the other two ratios, the discount rates do not change significantly. The reason is that when the value of this ratio is high relative to the average demand, as it is in this example, the discount rates are not very sensitive to changes in the value of this ratio.

More importantly, the discount rates in Table 4.9 are lower than the discount rates in Table 4.7 (discount rates when both revenues and costs are considered). The difference in the discount rates of the two tables increases with a decrease in price and an increase in the beta of demand. Given the significant differences in the discount rates, it is easy to see that if discount rates are not adjusted for the method used, the net present values could be biased, which could lead to incorrect decisions.

Suppose for the moment that the cost savings from the FMS are discounted at 20%. [18] Given our estimates that the risk-free rate is 6.0% and the market risk premium is 8.5%, a 20% discount rate implies that the beta of cost savings is 1.65. Substituting the cost savings from the FMS (savings of $1.48 million in fixed costs and of $1.28 in variable costs per unit) in our model for computing the appropriate beta for cost savings (equation (4.9)), a beta of cost savings of 1.65 implies that the beta of demand is 5.2. Suppose that the selling price is $13.5. Then using equation (4.4) with the cost structure of the FMS, the beta of

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on the ratio of fixed to variable costs or the ratio of the savings in fixed costs to the savings in variable costs.

[18] The firm actually used a higher discount rate than 20%.

the firm is 8.4. These values of the beta of demand and the beta of the firm are much higher than empirically observed betas. For example, Rosenberg and Guy (1976) estimate that betas for stocks of industries range from a high of 1.8 for air transport to a low of 0.35 for gold. [19] Even if a discount rate of 10% is used to discount the cost savings from the FMS, the implied value of the beta of demand is 1.5. A beta of demand of 1.5 implies that the beta of the firm with the FMS is 2.4. These values, although still high, are more consistent with the empirically observed values of beta.

It is common to find firms using discount rates of 20% to 30% for evaluating new technologies on the basis of cost savings (Hayes and Garwin (1982), p.76, and Sprow (1987), p.54). Based on the example just discussed and the empirical evidence on the industry betas, one must question the appropriateness of using such high discount rates. Since such high disount rates are being used, it is not surprising that firms are finding it difficult to financially justify new technologies on the basis of cost savings.

If the choice of technology has no impact on price and demand, as assumed in this chapter, then all three methods of evaluating technologies should give the same net present values and should lead to the same technology choice decision. However, for this to happen it is important that the relevant expected cash flows from each method are discounted at the discount rate appropriate for that method. Using any other discount rates will not give the same net present values from

[19] These estimates of betas include the effect of financial leverage. The higher the financial leverage, the higher is the beta of the firm's stock. Another set of estimates of industry betas are given in Brealey and Myers (1981, p.167). These estimates of industry betas range from a high of 1.49 for electronic components to a low of 0.46 for electric utilities. These estimates are of asset betas. The effect of financial leverage on beta has been removed. The source of these estimates is U.S. Federal Energy Regulatory Commission, Testimony of Gerald A. Pogue, Williams Pipe Co., Docket Nos. OR79-1, et al., p.74.

all three methods and the choice of technology can change depending on the method used. I illustrate this point using the data on the cost structures of the conventional technology and the flexible manufacturing system (FMS) from my case study (Table 4.4). I compare the technology choice decision from the method that considers both revenues and costs, and the total costs method when (1) the discount rates are adjusted for the method used, and (2) the discount rates for evaluating the two technologies on the basis of total costs are not adjusted for the method used, but are based on the method that considers both revenues and costs. For illustrative purposes assume the following: (1) Selling price is $13.5 per unit, (2) both technologies will last for one year, and (3) the beta of demand is 0.5.

Panel A of Table 4.10 gives the present values when the relevant expected cash flows from each of the two methods are discounted at the discount rate (in parentheses) appropriate for that method (see Tables 4.7 and 4.9). If the conventional technology is replaced by the FMS, both methods estimate that the present value of the firm increases by $2.03. Since the net present value of replacing the conventional technology with the FMS is the increase in the present value less the initial investment, both methods give the same net present values and will lead to the same technology choice decision.

Panel B of Table 4.10 gives the present values when the discount rates for evaluating the two technologies on the basis of total costs are not adjusted for the method used, but are based on the method that considers both revenues and costs. If the conventional technology is replaced by the FMS, the estimates of the increase in the present value of the firm are different from the two methods. Hence the net present value of replacing the conventional technology with the

Table 4.10

Present Values (PV) of the Expected Cash Flows from the Conventional Technology and the Flexible Manufacturing System (FMS).

A. Estimates of Present Values of the Two Technologies When Discount Rates are Adjusted for the Method Used.*

Technology	Correct Estimate of Present Values Based On Revenues and Costs Method	Correct Estimate of Present Values Based On Total Costs Method
Conventional Technology	1.30 (22.0 %)	5.37 (7.4 %)
FMS	3.33 (12.8 %)	3.34 (7.5 %)
Increase in Present Value If Conventional Technology is Replaced by FMS	2.03	2.03

B. Estimates of Present Values When Discount Rates for Evaluating the Two Technologies on the Basis of Total Costs are Not Adjusted for the Method Used, but are Based on the Method that Considers Both Revenues and Costs*

Technology	Correct Estimate of Present Values Based On Revenues and Costs Method	Incorrect Estimate of Present Values Based On Total Costs Method
Conventional Technology	1.30 (22.0 %)	4.72 (22.0 %)
FMS	3.33 (12.8 %)	3.18 (12.8 %)
Increase in Present Value If Conventional Technology is Replaced by FMS	2.03	1.54

* Present values in millions of dollars.
Discount rates are given in parentheses.
Present values computed under the following assumptions: (1) selling price is $13.5 per piece, (2) both technologies will last for one year, and (3) the beta of demand is 0.5.

FMS will not be the same from both methods and the choice of technology can change depending on the method. To illustrate this point, suppose that the initial investment for the FMS is $1.9 million. Based on the data in Panel B, it is easy to see that if the method that considers both revenues and costs is used, then the firm should invest in the FMS. If the method that considers total costs is used, then the firm should not invest in the FMS. On the other hand, the data in Panel A shows that both methods give the same decision that the firm should invest in the FMS.

To apply the models discussed here, the critical parameters that need to be estimated are the cost structure of a technology and the beta of demand. For the most part, the cost structure of the existing technology can be estimated from the firm's accounting data and that of new technologies from internal studies that are usually undertaken when investments in new technologies are considered.

Estimating the beta of demand is more difficult. A possible place to start with is the beta of the firm's common stock. [20] The stock beta reflects the effect of financial leverage, operating leverage, and the beta of demand. Hence, to estimate the beta of demand, the stock beta must be adjusted for the effect of financial and operating leverage.

Whenever a firm issues debt, the commitment to fixed debt charges creates financial leverage. The higher the financial leverage, the higher is the beta of the firm's stock. The stock beta can be adjusted for the effect of financial leverage using a technique proposed by Hamada (1972). This adjusted beta is commonly called the "asset" beta. The asset beta reflects the effect of operating leverage

[20] Beta is one of the most widely estimated parameters in finance. Numerous academic papers have estimated betas for individual stocks, portfolio of stocks, and bonds. For practitioners, there are beta 'books' and beta services. Betas are mostly estimated from time series of past stock returns, using the market model. Jensen (1972) and Fama (1976) provides a useful discussion of some of the variations on the simplest form of the market model that have been used to estimate betas.

and the beta of demand. Equation (4.4) can be used to adjust the asset beta for operating leverage, to obtain the beta of demand.

Once an estimate of the beta of demand is available, the beta of the technology can be obtained by adjusting the beta of demand for the cost structure of the technology. Translating this beta into a discount rate requires estimates of the risk-free rate of return and the market risk premium. For the most part, the risk-free rate is an observable parameter, so that it need not be estimated. A common approach for estimating the market risk premium is to use the average of the time series of realized market risk premiums. Merton (1980) observes that this approach does not account for the effect of changes in the level of market risk on the market risk premium. He presents models for estimating the market risk premium that account for the changes in the level of market risk. French, Schwert, and Stambaugh (1987) provide evidence that the market risk premium is positively related to the market risk.

4.6 Summary

The analysis presented in this chapter has three main results. First, the cost structure of new technologies is different from that of conventional technologies. As suggested by the case-style evidence presented here, new technologies require a higher initial investment than conventional technologies, but have lower fixed operating costs per period and lower variable costs per unit than conventional technologies. Second, since the cost structure of a technology affects the risk of the firm, the discount rate for evaluating a technology should be adjusted for its cost structure. If new technologies have lower fixed costs and lower variable costs than conventional technologies, then a lower discount rate should be used to discount the cash flows from new technologies. As shown by the example from a case study, adjusting the discount rate for the cost structure of the technology results in significantly different net present values, which could impact the technology choice decision. Third, the appropriate discount rate for evaluating technologies solely on the basis of costs is different from the discount rate when both revenues and costs are considered. In many cases, the discount rates when only costs are considered should be lower than when both revenues and costs are considered.

There are at least three directions in which extensions of the analysis of this chapter could prove useful. First, is to develop models which estimate the value of the many strategic benefits of new technology, such as higher flexibility, reduced lead times, better quality, and improved customer response times. These benefits not only reduce operating costs, but also result in higher market share or higher prices for the firm's products. The possible gains in market share and price increases will also depend on the industry demand and supply conditions. Ceteris paribus, higher market share and higher prices reduce the risk of the firm. This can be easily seen from the models developed in this chapter. However, the critical issue is to estimate possible gains in market share and price increases when firms adopt new technolgies.

A second direction is to consider the effect of the initial investment on the risk of the firm. There are at least two ways of considering this. First, when salvage value of the technology is not zero, the decision to liquidate or replace the technology will affect the terminal period's cash flows, thereby affecting the risk of the firm. An interesting issue here is to see if the salvage value of new and conventional technologies are different. Second, when taxes are considered, the depreciation tax shields affects the cash flows of the firm, and therefore the risk of the firm.

The third direction is to extend the analysis to consider multiperiod models. [21] An interesting issue here is to consider the interaction between production, inventory and technology choice decisions of the firm, and to see how these interactions affect the risk of the firm. For this purposes, the modeling approach proposed by Cohen and Halperin (1986) could be useful. They, however, do not address the issue of risk.

[21] Multiperiod versions of the capital asset pricing model are available (Merton (1973)) and have been used for multiperiod capital budgeting problems by Brennan (1973), Bogue and Roll (1974), and Myers and Turnbull (1975), among others. However, the use of capital asset pricing model for multiperiod capital budgeting problems has been the subject of extensive theoretical discussion (see Fama (1977) and Constantinides (1980)).

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APPENDIX A.

Relation Between the Beta of the Firm and its Inventory

Consider a firm that buys and sells a single product, which has stochastic demand. Let P be the selling price per unit, C be the purchase cost per unit, and h be the holding cost per unit. Assume that the inventory held by the firm is sufficient to meet all possible demand without any backordering. Let I be the average level of inventory held by the firm. The total inventory holding cost incurred during the period is the product of the holding cost per unit and the average inventory level. The firm exists for a single period and all cash flows occur at the end of the period. Let $\tilde{X}$ be the uncertain cash flows of the firm. Then $\tilde{X}$ can be written as

$$\tilde{X} = (P - C)\tilde{D} - hI. \qquad (A.1)$$

From the CAPM (see equation (2.5)), we have the value of the firm, $V(\tilde{X})$, as

$$V(\tilde{X}) = \frac{(P - C)(\bar{D} - \lambda \mathrm{Cov}(\tilde{D}, \tilde{R}_m)) - hI}{(1 + R_F)}, \qquad (A.2)$$

where $\bar{D}$ is the expected value of demand, λ is the market price per unit of risk, $\mathrm{Cov}(\tilde{D}, \tilde{R}_m)$ is the covariance of demand with the market return, $\tilde{R}_m$, and R_F is the risk-free rate of return. Note that the covariance of the fixed inventory holding cost, hI, with the market return, $\tilde{R}_m$, is equal to zero.

The uncertain rate of return, $\tilde{R}$, on the firm's common stock is

$$\tilde{R} = \frac{\tilde{X}}{V(\tilde{X})} = \frac{[(P-C)\tilde{D} - hI](1+R_F)}{(P-C)(\bar{D} - \lambda \text{Cov}(\tilde{D}, \tilde{R}_m)) - hI}. \quad (A.3)$$

The risk of the firm, measured by the beta, β, is defined as

$$\beta = \frac{\text{Cov}(\tilde{R}, \tilde{R}_m)}{\sigma_m^2}, \quad (A.4)$$

where σ_m^2 is the variance of the market return. Substituting expression (A.3) for $\tilde{R}$ in $(A.4)$ gives

$$\beta = \frac{(1+R_F)(P-C)\text{Cov}(\tilde{D}, \tilde{R}_m)}{\{(P-C)(\bar{D} - \lambda \text{Cov}(\tilde{D}, \tilde{R}_m)) - hI\}\sigma_m^2}. \quad (A.5)$$

Dividing both the numerator and the denominator in $(A.5)$ by $(P-C)(\bar{D} - \lambda \text{Cov}(\tilde{D}, \tilde{R}_m))$ yields

$$\beta = \frac{(1+R_F)\text{Cov}(\tilde{D}, \tilde{R}_m)}{\sigma_m^2(\bar{D} - \lambda \text{Cov}(\tilde{D}, \tilde{R}_m))(1 - \frac{hI}{(P-C)(\bar{D} - \lambda \text{Cov}(\tilde{D}, \tilde{R}_m))})}. \quad (A.6)$$

It can be shown that the beta of demand, β_D, is

$$\beta_D = \frac{(1+R_F)\text{Cov}(\tilde{D}, \tilde{R}_m)}{\sigma_m^2(\bar{D} - \lambda \text{Cov}(\tilde{D}, \tilde{R}_m))}. \quad (A.7)$$

Substituting for β_D in $(A.6)$, we have

$$\beta = \frac{\beta_D}{1 - \frac{hI}{(P-C)(\bar{D} - \lambda \text{Cov}(\tilde{D}, \tilde{R}_m))}}. \quad (A.8)$$

APPENDIX B.

Proofs

A. **Proof of Lemma 3.2.** If $\pi_i > 0$ for $i = 0, 1, 2 \ldots$, then $MEU(s, \lambda) > MEU(s+1, \lambda)$.

Proof: For fixed p it is easy to see that for $s > \hat{s}$, $MEU(s, \lambda)$ is decreasing in s since $M(s+1) > p$ and marginal cost is increasing. For $s \leq \hat{s}$,

$$\begin{aligned} MEU(s,\lambda) = & \sum_{i=0}^{s}(U(i,s+1) - U(i,s))e^{-\lambda}\ \frac{\lambda^i}{i!} \\ & + (U(s+1,s+1) - U(s,s))(1 - F(s)) \end{aligned} \quad (B.1)$$

and

$$\begin{aligned} MEU(s+1,\lambda) = & \sum_{i=0}^{s+1}(U(i,s+2) - U(i,s+1))e^{-\lambda}\ \frac{\lambda^i}{i!} \\ & + (U(s+2,s+2) - U(s+1,s+1))(1 - F(s+1)) \\ = & \sum_{i=0}^{s}(U(i,s+2) - U(i,s+1))e^{-\lambda}\ \frac{\lambda^i}{i!} \\ & + (U(s+1,s+2) - U(s+1,s+1))e^{-\lambda}\ \frac{\lambda^{s+1}}{s+1!} \\ & + (U(s+2,s+2) - U(s+1,s+1))(1 - F(s+1)) \end{aligned} \quad (B.2)$$

From Lemma 3.1 we have:

$$U(i,s+1) - U(i,s) > U(i,s+2) - U(i,s+1) \quad for \quad i = 0,1,\ldots,s \quad (B.3)$$

$$U(s+1,s+2) - U(s+1,s+1) < 0 \tag{B.4}$$

$$U(s+1,s+1) - U(s,s) > U(s+2,s+2) - U(s+1,s+1) \tag{B.5}$$

Also

$$(1 - F(s)) > (1 - F(s+1)) \tag{B.6}$$

On comparing $(B.1)$ and $(B.2)$ term by term, we have

$MEU(s, \lambda) > MEU(s+1, \lambda)$.

B. Proof that for fixed s, $EU(s, \lambda)$ is a concave function of λ.

The first derivative of $EU(s, \lambda)$ for fixed s, is

$$\begin{aligned}
\frac{\partial EU(s,\lambda)}{\partial \lambda} &= \sum_{i=0}^{s} -U(i,s)e^{-\lambda}\frac{\lambda^i}{i!} \\
&+ \sum_{i=1}^{s} U(i,s)e^{-\lambda}\frac{\lambda^{i-1}}{i-1!} \\
&+ U(s,s)e^{-\lambda}\frac{\lambda^s}{s!} \\
&= \sum_{i=0}^{s-1} -U(i,s)e^{-\lambda}\frac{\lambda^i}{i!} \\
&+ \sum_{i=0}^{s-1} U(i+1,s)e^{-\lambda}\frac{\lambda^i}{i!} \\
&= \sum_{i=0}^{s-1} (U(i+1,s) - U(i,s))e^{-\lambda}\frac{\lambda^i}{i!}
\end{aligned} \tag{B.7}$$

For $i \leq s-1$, $\quad U(i+1,s) - U(i,s) > 0$, because $p(i+1) - C(s) > pi - C(s)$.

Therefore, we can see that $\frac{\partial EU(s,\lambda)}{\partial \lambda} > 0$.

The second derivative of $EU(s, \lambda)$ is

$$\begin{aligned} \frac{\partial^2 EU(s,\lambda)}{\partial \lambda^2} &= -\sum_{i=0}^{s-1}(U(i+1,s) - U(i,s))e^{-\lambda}\ \frac{\lambda^i}{i!} \\ &+ \sum_{i=1}^{s-1}(U(i+1,s) - U(i,s))e^{-\lambda}\ \frac{\lambda^{i-1}}{i-1!} \\ &= -\sum_{i=0}^{s-1}(U(i+1,s) - U(i,s))e^{-\lambda}\ \frac{\lambda^i}{i!} \\ &+ \sum_{i=0}^{s-2}(U(i+2,s) - U(i+1,s))e^{-\lambda}\ \frac{\lambda^i}{i!} \end{aligned} \qquad (B.8)$$

From the concavity of $U(P)$, it follows that $U(i+1,s) - U(i,s) > U(i+2,s) - U(i+1,s)$ for $i \leq s-2$. Therefore, we can easily see that $\frac{\partial^2 EU(s,\lambda)}{\partial \lambda^2} < 0$.

APPENDIX C.

Relation Between the Beta of the Firm and its Cost Structure

This appendix derives the expression for the beta of the following risky cash flow, $\tilde{X}$, from our technology choice model:

$$\tilde{X} = (P - C)\tilde{D} - F. \qquad (C.1)$$

Assume that risky cash flows are valued in perfect capital markets according to the Sharpe-Lintner Capital Asset Pricing Model (CAPM), such that the equilibrium value $V(\tilde{X})$ of any risky cash flow $\tilde{X}$ is:

$$V(\tilde{X}) = \frac{E(\tilde{X}) - \lambda \mathrm{Cov}(\tilde{X}, \tilde{R}_m)}{1 + R_f}, \qquad (C.2)$$

where R_f is the single period risk-free rate of return; $\tilde{R}_m$ is the single period rate of return on the portfolio that consists of all risky assets in the market, and has an expected value of $E(\tilde{R}_m)$ and a standard deviation of σ_m; $\lambda = [E(\tilde{R}_m) - R_f]/\sigma_m^2$ is the market price per unit of risk; $\mathrm{Cov}(\tilde{X}, \tilde{R}_m)$ is the covariance between $\tilde{X}$ and $\tilde{R}_m$; and $E(\tilde{X})$ is the expected value of the cash flow.

The numerator on the right hand side of $(C.2)$ is called the certainty equivalent of the uncertain cash flow, $\tilde{X}$. The certainty equivalent of the cash flow is the expected cash flow, $E(\tilde{X})$, minus a risk discount given by the product of λ, the market price of risk, and the risk of the cash flow, given by the covariance of the cash flow with the market return. Using equation $(C.2)$ to value the cash flows in $(C.1)$, we have the value of the firm, $V(\tilde{X})$, as

$$V(\tilde{X}) = \frac{(P-C)(\bar{D} - \lambda \mathrm{Cov}(\tilde{D}, \tilde{R}_m)) - F}{(1+R_f)}, \tag{C.3}$$

where $\bar{D}$ is the expected value of demand, and $\mathrm{Cov}(\tilde{D}, \tilde{R}_m)$ is the covariance of demand with the market return, $\tilde{R}_m$. Note that the covariance of the fixed operating cost, F, with the market return, $\tilde{R}_m$, is equal to zero.

The uncertain rate of return, $\tilde{R}$, on the firm's common stock is

$$\tilde{R} = \frac{\tilde{X}}{V(\tilde{X})} = \frac{((P-C)\tilde{D} - F)(1+R_f)}{(P-C)(\bar{D} - \lambda \mathrm{Cov}(\tilde{D}, \tilde{R}_m)) - F}. \tag{C.4}$$

The risk of the firm, measured by the beta, β, is defined as

$$\beta = \frac{\mathrm{Cov}(\tilde{R}, \tilde{R}_m)}{\sigma_m^2}, \tag{C.5}$$

where σ_m^2 is the variance of the market return. Substituting expression $(C.4)$ for $\tilde{R}$ in $(C.5)$ we have

$$\beta = \frac{(1+R_f)(P-C)\mathrm{Cov}(\tilde{D}, \tilde{R}_m)}{\{(P-C)(\bar{D} - \lambda \mathrm{Cov}(\tilde{D}, \tilde{R}_m)) - F\}\sigma_m^2}. \tag{C.6}$$

Dividing both the numerator and the denominator in $(C.6)$ by $(P-C)(\bar{D} - \lambda \mathrm{Cov}(\tilde{D}, \tilde{R}_m))$ yields

$$\beta = \frac{(1+R_f)\mathrm{Cov}(\tilde{D}, \tilde{R}_m)}{\sigma_m^2(\bar{D} - \lambda \mathrm{Cov}(\tilde{D}, \tilde{R}_m))(1 - \frac{F}{(P-C)(\bar{D} - \lambda \mathrm{Cov}(\tilde{D}, \tilde{R}_m))})}. \tag{C.7}$$

It can be shown that the beta of demand, β_D, is

$$\beta_D = \frac{(1+R_f)\mathrm{Cov}(\tilde{D}, \tilde{R}_m)}{\sigma_m^2(\bar{D} - \lambda \mathrm{Cov}(\tilde{D}, \tilde{R}_m))}. \tag{C.8}$$

Substituting for β_D in $(C.7)$, we have

$$\beta = \frac{\beta_D}{1 - \frac{F}{(P-C)(\bar{D} - \lambda \mathrm{Cov}(\tilde{D}, \tilde{R}_m))}}. \tag{C.9}$$